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Solving Pythagorean Fuzzy Assignment Problems in Management: A Framework Based on Spherical Distance Measures

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ABSTRACT

In practical scenarios, uncertainty often arises from measurement limitations or incomplete information, making it impossible to obtain exact values for key variables. Additionally, decision-makers may struggle to articulate precise judgments under constraints such as time or limited knowledge. To better capture this ambiguity, fuzzy-based frameworks allow individuals to express their assessments in more flexible terms. Among these, the Pythagorean fuzzy set offers a broader descriptive range than intuitionistic fuzzy sets for representing degrees of membership and non-membership. This paper introduces two approaches based on positive and negative ideal solutions to solve assignment problems under Pythagorean fuzzy conditions by applying a spherical distance measure and a new scoring method. The effectiveness of the proposed technique is illustrated through numerical examples.

1. Introduction

An assignment problem is a type of linear programming problem (LPP) concerned with the optimal allocation and scheduling of resources. Such problems arise because available resources often differ in their efficiency for performing various tasks. Classical assignment models assume that decision-makers know the exact costs associated with each assignment; however, in real-world situations, these

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values are frequently imprecise.

To address uncertainty in practical problems, Zadeh [1] introduced the concept of fuzzy sets (FS). Building on this idea, several extensions of fuzzy sets have been developed to better capture vagueness and imprecision. One such extension is the intuitionistic fuzzy set (IFS), proposed by Atanassov [2], which incorporates both membership and nonmembership degrees for each element.

Yager [3] introduced a Pythagorean fuzzy set (PFS). Yager overcomes the situation when sum of membership degree and nonmembership degree greater than 1. PFS is an extension of IFS with the condition that the square sum of the membership degree and the nonmembership degree is less than or equal to 1. The concept of Pythagorean fuzzy sets (PFS) gives the larger preference domain for decision makers (DM). DMs can define their support and against the degree of membership as $\alpha = 0.7$, $\beta(x) = 0.5$. In this case, $0.7 + 0.5 > 1$ is not valid in IFS but squaring $0.7^2 + 0.5^2 < 1$ implies the Pythagorean fuzzy set is more suitable than the intuitionistic fuzzy set. Tahir studied on Pythagorean Soft Sets and Hypersoft Sets in [4]. Adak et al. [5, 6] investigated loan prediction under pythagorean fuzzy environment. Asif et al. [7] utilized Hamacher Aggregation Operators for solving MCDM problems for PFSs. Several researcher investigated some important decision making problems under uncertain environment [8–11].

Gurukumaresan et al. [12] employed the centroid method to solve fuzzy assignment problems. Kumar and Gupta [13] addressed fuzzy assignment problems and fuzzy travelling salesman problems using various membership functions. Thakre et al. [14] applied fuzzy assignment techniques to staff placement in LIC. Roseline and Amirtharaj [15] solved intuitionistic fuzzy assignment problems using a ranking approach for intuitionistic fuzzy numbers (IFNs). Mukherjee and Basu [16] tackled assignment problems under the intuitionistic fuzzy set (IFS) framework by applying similarity measures and score functions. Furthermore, Kumar and Bajaj [17] introduced interval-valued intuitionistic fuzzy assignment problems and proposed solutions based on similarity measures and score functions. In this work, we develop a methodology to solve assignment problems involving Pythagorean fuzzy values.

The paper is organized as follows: Section 2 provides essential background on fuzzy sets (FS), intuitionistic fuzzy sets (IFS), Pythagorean fuzzy sets (PFS), and operations on Pythagorean fuzzy numbers. Section 3 introduces the proposed spherical distance measure and the associated score function. Section 4 presents the methodology for solving Pythagorean fuzzy assignment problems (PFAP) using the spherical distance measure. A numerical example discuss in Section 5. Section 6 offers a comparative study along with concluding remarks.

2. Preliminaries and Definition

In this section, we recall some basic notions such as the intuitionistic fuzzy sets and the Pythagorean fuzzy sets. Also, we include some elementary aspects that are necessary for this paper. Let X be a set of finite universal sets. An intuitionistic fuzzy set (IFS) I in X is an expression having the form

$$I = \{\langle x, \alpha_I(x), \beta_I(x)\rangle : x \in X\},$$

where the functions $\alpha_I(x)$ and $\beta_I(x)$ are the degree of membership and the degree of non-membership of the element $x \in X$ respectively. Also $\alpha_I : X \to [0,1]$, $\beta_I : X \to [0,1]$ and $0 \leq \alpha_I(x) + \beta_I(x) \leq 1$, for all $x \in X$.

The degree of indeterminacy $\pi_I(x) = 1 - \alpha_I(x) - \beta_I(x)$. A Pythagorean fuzzy set P in a finite universe of discourse X is given by

$$P = \{\langle x, \alpha_P(x), \beta_P(x)\rangle | x \in X\},$$

where $\alpha_P(x) : X \to [0,1]$ denotes the degree of membership and $\beta_P(x) : X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set A respectively with the condition that

$0 \leq (\alpha_P(x))^2 + (\beta_P(x))^2 \leq 1$.

The degree of indeterminacy $\gamma_P(x) = \sqrt{1 - (\alpha_P(x))^2 - (\beta_P(x))^2}$.

2.1 Some Operations on Pythagorean Fuzzy Numbers

Here we discussed some operations on Pythagorean fuzzy numbers and Pythagorean fuzzy sets those are used in the rest of the paper.

Given three Pythagorean fuzzy numbers (PFNs) $p = \langle \alpha, \beta \rangle$, $p_1 = \langle \alpha_1, \beta_1 \rangle$ and $p_2 = \langle \alpha_2, \beta_2 \rangle$. The basic operations can be defined as follows:

(i) $\bar{p} = \langle \beta, \alpha \rangle$

(ii) $p_1 \cup p_2 = \langle \max\{\alpha_1, \alpha_2\}, \min\{\beta_1, \beta_2\} \rangle$

(iii) $p_1 \cap p_2 = \langle \min\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\} \rangle$

Let $p_1 = \langle 0.7, 0.3 \rangle$ and $p_2 = \langle 0.8, 0.5 \rangle$, then
$p_1 \cup p_2 = \langle 0.8, 0.3 \rangle$, $p_1 \cap p_2 = \langle 0.7, 0.5 \rangle$ and $\bar{p_1} = \langle 0.3, 0.7 \rangle$. Let $p = \langle \alpha, \beta \rangle$ be a pythagorean fuzzy number. The score function of p is defined as

$$S(p) = (\alpha)^2 - (\beta)^2,$$

where $S(p) \in [-1, 1]$.

Let $p = \langle \alpha, \beta \rangle$ be a pythagorean fuzzy number. The accuracy function of p is defined as

$$H(p) = (\alpha)^2 + (\beta)^2,$$

where $H(p) \in [0, 1]$. Peng and Dai (2017) suggested another score formula of PFNs by the exponential function. Let $p = \langle \alpha, \beta \rangle$ be a pythagorean fuzzy number. The exponential score function S_{pd} of p is defined as

$$S_{pd}(p) = \frac{e^{\alpha^2 - \beta^2}}{\pi^2 + 1}.$$

Let us consider a PFN $p = \langle 0.7, 0.4 \rangle$. Then the values of different score functions are $S(p) = 0.33$, $H(p) = 0.65$, $S_{pd}(p) = 0.127$.

3. Spherical Distance Measurement Method for PFNs

Let $p = \langle \alpha, \beta \rangle$ be a Pythagorean fuzzy number satisfying $0 \leq \alpha^2 + \beta^2 \leq 1$,and let the hesitation degree be $\gamma = \sqrt{1 - \alpha^2 - \beta^2}$, so that $\alpha^2 + \beta^2 + \gamma^2 = 1$.

From this relation, the triplet (α, β, γ) can be interpreted as a point lying on the surface of a unit sphere centered at the origin. This geometric interpretation motivates the definition of a spherical distance between two Pythagorean fuzzy numbers on this restricted spherical surface.

On a spherical surface, the shortest path between two points is the arc length of the great circle passing through them.

Let A and C be two points on the spherical surface with co-ordinate (x_1, y_1, z_1) and (x_2, y_2, z_2), then the spherical distance between these two points is defined as

$$D_{SP}(A, C) = \frac{2}{\pi} \arccos \left\{ 1 - \frac{1}{2} \left[(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 \right] \right\} \quad (1)$$

Incorporated this expression, the spherical distance between two Pythagorean fuzzy numbers are defined as follows: Let $p_1 = \langle \alpha_1, \beta_1 \rangle$ and $p_1 = \langle \alpha_2, \beta_2 \rangle$ be two Pythagorean fuzzy numbers with hesitation function γ_1 and γ_2 respectively. Then the spherical distance between these two Pythagorean fuzzy numbers is

$$D_{SP}(p_1, p_2) = \frac{2}{\pi} \arccos \left\{ 1 - \frac{1}{2}[(\alpha_1 - \alpha_2)^2 + (\beta_1 - \beta_2)^2 + (\gamma_1 - \gamma_2)^2] \right\} \tag{2}$$

To get the distance value in between $[0, 1]$ the factor $\frac{2}{\pi}$ is introduced.
Since, $\alpha_1^2 + \beta_1^2 + \gamma_1^2 = 1$ and $\alpha_2^2 + \beta_2^2 + \gamma_2^2 = 1$, so after simplifying the equation (2), we have

$$D_{SP}(p_1, p_2) = \frac{2}{\pi} \arccos [\alpha_1 \alpha_2 + \beta_1 \beta_2 + \gamma_1 \gamma_2] \tag{3}$$

Now, we define the spherical and normalized distances between two Pythagorean fuzzy sets. Let $P = \{x_i, < \alpha_P(x_i), \beta_P(x_i) >: x_i \in X\}$ and $Q = \{x_i, < \alpha_Q(x_i), \beta_Q(x_i) >: x_i \in X\}$ of the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, then their spherical and normalized spherical distances are
Spherical Distance:

$$D_{SP}(P, Q) = \frac{2}{\pi} \sum_{i=1}^{n} \arccos [\alpha_P(x_i)\alpha_Q(x_i) + \beta_P(x_i)\beta_Q(x_i) + \gamma_P(x_i)\gamma_Q(x_i)] \tag{4}$$

where $0 \leq D_{SP}(P, Q) \leq n$.
Normalized Spherical Distance:

$$D_{NS}(P, Q) = \frac{2}{n\pi} \sum_{i=1}^{n} \arccos [\alpha_P(x_i)\alpha_Q(x_i) + \beta_P(x_i)\beta_Q(x_i) + \gamma_P(x_i)\gamma_Q(x_i)] \tag{5}$$

where $0 \leq D_{NSP}(P, Q) \leq 1$.

Let $p_1 = \langle 0.9, 0.2 \rangle$ and $p_2 = \langle 0.7, 0.3 \rangle$ be two pythagorean fuzzy numbers. Then the spherical distance between p_1 and p_2 is

$$\begin{aligned} D_{SP}(p_1, p_2) &= \frac{2}{\pi} \arccos \left[(0.9 \times 0.7) + (0.2 \times 0.3) + (\sqrt{1 - 0.9^2 - 0.2^2} \times \sqrt{1 - 0.7^2 - 0.3^2}) \right] \\ &= 0.2198 \end{aligned}$$

4. Pythagorean Fuzzy Assignment Problem

In this section, we introduce the assignment problem involving Pythagorean fuzzy numbers and present two methodologies for solving Pythagorean fuzzy assignment problems.

In classical assignment problems, cost values are typically assumed to be deterministic. However, in real-world situations, it is often difficult to determine the exact cost due to uncertainty and imprecision. Consider an assignment problem involving the allocation of n workers to m tasks, where each worker can perform any task with varying efficiency in terms of cost, time, or other resources. Let c_{ij} denote the Pythagorean fuzzy cost coefficient associated with assigning the i-th task to the j-th worker, and let x_{ij} represent the decision variable for this assignment.

Our objective is to minimize the total cost by optimally allocating tasks to available workers. Under such uncertain conditions, we compute a preference value that reflects the relative suitability of each assignment. Based on these preference values, we determine the desirability of assigning the j-th task to the i-th worker in terms of a composite relative similarity to an ideal solution. Consequently, each c_{ij} is replaced by its corresponding composite relative degree.

The mathematical formulation of Pythagorean fuzzy assignment problem as follows:

$$\min Y = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\text{subject to,} \quad \sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1, 2, \ldots, n; \quad x_{ij} \in \{0, 1\}.$$

where $c_{ij} = (\alpha_{ij}, \beta_{ij})$;
α_{ij}: Degree of satisfaction (or low cost) for that assignment.
β_{ij}: Degree of dissatisfaction (or high cost) for that assignment.

Positive and negative ideal solution plays a crucial role in pythagorean fuzzy assignment problems. Here, we introduced two methodologies for calculating positive and negative ideal solutions. First one is based on max and min concept on membership and non-membership value. The second one is based on exponential score function. Depending on these, two methodologies have been introduced to solve pythagorean fuzzy assignment problems. Methodologies are discussed as follows.

4.1 *Proposed Methodology*

Step-1: First consider the Pythagorean fuzzy assignment problem through a cost matrix $G = (c_{ij})_{m\times n}$, where $c_{ij} = \langle \alpha_{ij}(x), \beta_{ij}(x)\rangle$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ are pythagorean fuzzy numbers.
Step-2: Check whether the assignment problem is balanced. If it is not balanced, introduce appropriate dummy variables to convert it into a balanced assignment problem.
Step-3: Pythagorean fuzzy positive ideal solution and pythagorean fuzzy negative ideal solution are calculated by two ways.
Case-I:Column pythagorean fuzzy positive ideal solution as

$$a_j^+ = \langle \max\{\alpha_{ij}\}, \min\{\beta_{ij}\}\rangle, j = 1, 2, \ldots, n$$

and column wise pythagorean fuzzy negative ideal solution as

$$a_j^- = \langle \min\{\alpha_{ij}\}, \max\{\beta_{ij}\}\rangle, j = 1, 2, \ldots, n.$$

Similarly, row wise pythagorean fuzzy positive and negative ideal solution calculated.
Case-II: In this case, utilizing the formula $S_{pd} = \frac{e^{\alpha^2-\beta^2}}{\pi^2+1}$ first calculate exponential score value for each cost. The column wise pythagorean fuzzy positive and negative ideal solution are maximum value score value and minimum score value of the particular column respectively. Similarly, for the row wise positive nd negative ideal solution.
Step-4: Calculate the spherical distance measure of each cost value from pythagorean positive ideal solution and pythagorean negative ideal solution. Two new matrix are obtained as $\mathcal{D}_{SP}(C, C^+))$ and $\mathcal{D}_{SP}(C, C^-)$
Step-5: Column wise relative matrix are calculated by the formula

$$Q = \frac{\mathcal{D}_{SP}(C, C^+))}{\mathcal{D}_{SP}(C, C^+)) + \mathcal{D}_{SP}(C, C^-)} = (q_{ij})_{n\times n}$$

and row wise relative matrix

$$R = \frac{\mathcal{D}_{SP}(C, C^{+}))}{\mathcal{D}_{SP}(C, C^{+})) + \mathcal{D}_{SP}(C, C^{-})} = (r_{ij})_{n\times n}$$

Step-6: The composite matrix $[T]_{n\times n}$ is calculated as $T = Q \times R = q_{ij} \times r_{ij}$, the resultant matrix T represents the preference that j-th job is chosen by i-th person.

5. Illustrative Example

A company has 6 jobs $(\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_6)$ to be assigned to 6 machines $(\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5, \Upsilon_6)$. The costs of assigning each job to each machine are represented as pythagorean fuzzy numbers, as shown in the following table. The goal is to find the optimal assignment that minimizes the total cost. A PF cost matrix captures the ambivalence and nuance in decision-making more richly. For instance:

Table 1
PF Cost Matrix

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	$\langle 0.64, 0.45\rangle$	$\langle 0.82, 0.36\rangle$	$\langle 0.77, 0.52\rangle$	$\langle 0.91, 0.23\rangle$	$\langle 0.74, 0.67\rangle$	$\langle 0.56, 0.61\rangle$
Υ_2	$\langle 0.71, 0.57\rangle$	$\langle 0.66, 0.48\rangle$	$\langle 0.55, 0.70\rangle$	$\langle 0.81, 0.20\rangle$	$\langle 0.38, 0.81\rangle$	$\langle 0.45, 0.59\rangle$
Υ_3	$\langle 0.83, 0.42\rangle$	$\langle 0.69, 0.71\rangle$	$\langle 0.83, 0.29\rangle$	$\langle 0.71, 0.63\rangle$	$\langle 0.59, 0.38\rangle$	$\langle 0.50, 0.63\rangle$
Υ_4	$\langle 0.45, 0.68\rangle$	$\langle 0.85, 0.20\rangle$	$\langle 0.90, 0.21\rangle$	$\langle 0.63, 0.44\rangle$	$\langle 0.68, 0.48\rangle$	$\langle 0.57, 0.39\rangle$
Υ_5	$\langle 0.75, 0.36\rangle$	$\langle 0.91, 0.22\rangle$	$\langle 0.36, 0.82\rangle$	$\langle 0.73, 0.37\rangle$	$\langle 0.91, 0.33\rangle$	$\langle 0.62, 0.48\rangle$
Υ_6	$\langle 0.68, 0.38\rangle$	$\langle 0.63, 0.55\rangle$	$\langle 0.45, 0.59\rangle$	$\langle 0.84, 0.43\rangle$	$\langle 0.72, 0.41\rangle$	$\langle 0.77, 0.42\rangle$

$(0.9, 0.1)$: Excellent, clear choice.
$(0.6, 0.6)$: Highly ambiguous/neutral assignment.
$(0.7, 0.5)$: Leaning towards good, but with significant doubt

5.1 *Method-I*

Consider column wise pythagorean positive ideal solution as

$$a_j^{+} = \langle \max\{\alpha_{ij}\}, \min\{\beta_{ij}\}\rangle, j = 1, 2, \ldots, 6$$

and column wise pythagorean negative ideal solution as

$$a_j^{-} = \langle \min\{\alpha_{ij}\}, \max\{\beta_{ij}\}\rangle, j = 1, 2, \ldots, 6.$$

According to this formula, column wise positive and negative solutions are

$$a_j^{+} = \{\langle 0.83, 0.36\rangle, \langle 0.91, 0.20\rangle, \langle 0.90, 0.21\rangle, \langle 0.91, 0.20\rangle, \langle 0.91, 0.23\rangle, \langle 0.77, 0.39\rangle\}$$

$$a_j^{-} = \{\langle 0.45, 0.68\rangle, \langle 0.63, 0.71\rangle, \langle 0.36, 0.82\rangle, \langle 0.63, 0.63\rangle, \langle 0.38, 0.81\rangle, \langle 0.45, 0.63\rangle\}$$

Table 2
Columnwise positive ideal measure $\mathcal{D}_{SP}(C, C^{+})$

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.368	0.201	0.337	0.035	0.558	0.310
Υ_2	0.306	0.435	0.616	0.213	0.814	0.414
Υ_3	0.193	0.607	0.142	0.481	0.515	0.374
Υ_4	0.571	0.138	0	0.465	0.401	0.297
Υ_5	0.263	0.023	0.841	0.327	0.137	0.210
Υ_6	0.341	0.489	0.668	0.243	0.340	0.038

Table 3
Column wise negative ideal measure $\mathcal{D}_{SP}(C, C^{-})$

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.302	0.422	0.519	0.505	00554	0.133
Υ_2	0.328	0.352	0.225	0.480	0	0.054
Υ_3	0.512	0.142	0.724	0.160	0.554	0.063
Υ_4	0	0.590	0.841	0.266	0.462	0.284
Υ_5	0.442	0.573	0	0.304	0.759	0.227
Υ_6	0.383	0.284	0.335	0.316	0.543	0.415

Table 4
Column wise relative matrix S_1

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.549	0.322	0.393	0.064	0.501	0.699
Υ_2	0.482	0.552	0.732	0.307	1.0	0.884
Υ_3	0.273	0.810	0.163	0.750	0.481	0.855
Υ_4	1.0	0.189	0.0	0.636	0.464	0.511
Υ_5	0.373	0.038	1.0	0.449	0.152	0.480
Υ_6	0.470	0.632	0.666	0.434	0.385	0.083

Table 5
Row wise positive ideal measure $\mathcal{D}_{SP}(C, C^{+})$

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.449	0.187	0.324	0.0	0.558	0.567
Υ_2	0.410	0.320	0.579	0.0	0.772	0.550
Υ_3	0.170	0.563	0.0	0.397	0.350	0.463
Υ_4	0.693	0.117	0.0	0.442	0.391	0.513
Υ_5	0.295	0.0	0.844	0.325	0.149	0.478
Υ_6	0.289	0.315	0.531	0.075	0.212	0.123

Table 6
Row wise negative ideal measure $\mathcal{D}_{SP}(C, C^-)$

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.271	0.409	0.284	0.588	0.468	0.094
Υ_2	0.412	0.456	0.203	0.772	0.0	0.322
Υ_3	0.461	0.405	0.541	0.289	0.407	0.126
Υ_4	0.0	0.642	0.693	0.307	0.306	0.347
Υ_5	0.623	0.844	0.0	0.606	0.781	0.466
Υ_6	0.315	0.221	0.0	0.548	0.344	0.412

Table 7
Row wise relative matrix R_1

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.623	0.331	0.532	0.0	0.543	0.857
Υ_2	0.498	0.412	0.740	0.0	1.0	0.630
Υ_3	0.269	0.581	0.0	0.578	0.462	0..795
Υ_4	1.0	0.154	0.0	0.590	0.560	0.596
Υ_5	0.321	0.0	1.0	0.349	0.160	0.506
Υ_6	0.478	0.587	1.0	0.120	0.381	0.229

Preference matrix $T_1 = S_1 \times R_1$ and presented as

Table 8
Composite matrix T_1

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	1.167	0.953	1.173	0.523	1.183	1.437
Υ_2	1.822	1.369	2.548	1.059	1.820	2.234
Υ_3	1.193	1.131	2.080	0.807	1.856	1.760
Υ_4	1.746	0.788	1.646	0.598	1.357	1.706
Υ_5	1.247	1.064	0.858	0.953	1.161	1.593
Υ_6	1.383	0.909	1.185	0.785	1.531	1.802

Now, apply Hungarian algorithm to find the optimal assignment. The optimal assignment is $\Upsilon_1 \to \Gamma_6$, $\Upsilon_2 \to \Gamma_3$, $\Upsilon_3 \to \Gamma_5$, $\Upsilon_4 \to \Gamma_1$, $\Upsilon_5 \to \Gamma_2$, $\Upsilon_6 \to \Gamma_4$.

5.2 *Method-II*

In this model, first we calculate score value for each cost by utilizing exponential score function formula $S_{pd} = \frac{e^{\alpha^2-\beta^2}}{\pi^2+1}$.

Table 9
Score value of Cost

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.113	0.158	0.127	0.199	0.101	0.086
Υ_2	0.110	0.112	0.076	0.170	0.055	0.079
Υ_3	0.153	0.089	0.168	0.102	0.112	0.079
Υ_4	0.070	0.182	0.197	0.112	0.116	0.109
Υ_5	0.141	0.200	0.053	0.136	0.188	0.107
Υ_6	0.126	0.101	0.079	0.154	0.130	0.139

In this model, we consider the column wise pythagorean fuzzy positive and negative ideal solution as maximum value score value and minimum score value of the particular column. Similarly, for the row wise positive and negative ideal solution.

Column wise positive and negative solutions are

$$a_j^+ = \{\langle 0.83, 0.42\rangle, \langle 0.91, 0.22\rangle, \langle 0.90, 0.21\rangle, \langle 0.91, 0.23\rangle, \langle 0.91, 0.33\rangle, \langle 0.77, 0.42\rangle\}$$

$$a_j^- = \{\langle 0.45, 0.68\rangle, \langle 0.69, 0.71\rangle, \langle 0.55, 0.70\rangle, \langle 0.71, 0.63\rangle, \langle 0.38, 0.81\rangle, \langle 0.56, 0.61\rangle\}$$

Table 10
Column wise positive ideal measure $\mathcal{D}_{SP}(C, C^+)$

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.321	0.191	0.337	0.0	0.429	0.551
Υ_2	0.197	0.429	0.616	0.231	0.759	0.688
Υ_3	0.0	0.585	0.142	0.452	0.571	0.623
Υ_4	0.512	0.150	0	0.461	0.412	0.593
Υ_5	0.213	0.0	0.841	0.325	0.0	0.498
Υ_6	0.304	0.480	0.668	0.212	0.373	0.284

Table 11
Column wise negative ideal measure $\mathcal{D}_{SP}(C, C^-)$

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.302	0.486	0.298	0.452	0.554	0.293
Υ_2	0.328	0.500	0.0	0.506	0	0.322
Υ_3	0.512	0.0	0.502	0.0	0.554	0.262
Υ_4	0	0.648	0.616	0.387	0.462	0.544
Υ_5	0.442	0.585	0.225	0.370	0.759	0.447
Υ_6	0.383	0.445	0.262	0.239	0.543	0.559

Table 12
Column wise Relative matrix S_2

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.512	0.282	0.564	0.0	0.436	0.652
Υ_2	0.375	0.461	1.0	0.313	1.0	0.681
Υ_3	0.0	1.0	0.220	1.0	0.507	0.703
Υ_4	1.0	0.181	0.0	0.543	0.471	0.508
Υ_5	0.325	0.0	0.788	0.467	0.0	0.526
Υ_6	0.387	0.518	0.281	0.529	0.401	0.336

Table 13
Row wise positive ideal measure $\mathcal{D}_{SP}(C, C^+)$

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.449	0.187	0.324	0.0	0.558	0.567
Υ_2	0.410	0.320	0.579	0.0	0.772	0.550
Υ_3	0.170	0.563	0.0	0.397	0.350	0.463
Υ_4	0.693	0.117	0.0	0.442	0.391	0.513
Υ_5	0.295	0.0	0.844	0.325	0.149	0.478
Υ_6	0.289	0.315	0.531	0.075	0.212	0.123

Table 14
Row wise negative ideal measure $\mathcal{D}_{SP}(C, C^-)$

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.189	0.381	0.298	0.567	0.542	0.0
Υ_2	0.210	0.275	0.0	0.579	0.203	0.262
Υ_3	0.248	0.192	0.397	0.0	0.489	0.351
Υ_4	0.0	0.642	0.693	0.307	0.306	0.347
Υ_5	0.623	0.844	0.0	0.606	0.781	0.466
Υ_6	0.315	0.221	0.0	0.548	0.344	0.412

Table 15
Row wise Relative matrix R_2

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	0.703	0.329	0.520	0.0	0.507	1.0
Υ_2	0.661	0.537	1.0	0.0	0.791	0.677
Υ_3	0.406	0.745	0.0	1.0	0.417	0.584
Υ_4	1.0	0.154	0.0	0.590	0.560	0.483
Υ_5	0.321	0.0	1.0	0.349	0.160	0.506
Υ_6	0.470	0.595	1.0	0.0	0.430	0.285

Preference matrix $T_2 = S_2 \times R_2$ and presented as

Table 16
Composite matrix T_2

	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6
Υ_1	1.221	1.128	1.636	0.716	1.067	1.438
Υ_2	1.928	1.569	2.331	1.533	1.599	2.122
Υ_3	2.243	1.273	2.210	0.986	1.826	1.745
Υ_4	1.759	0.815	1.686	0.484	1.252	1.771
Υ_5	1.262	1.078	0.695	1.063	0.981	1.160
Υ_6	1.591	0.933	1.531	0.735	1.283	1.505

Similarly, apply Hungarian algorithm to find the optimal assignment. The optimal assignment is $\Upsilon_1 \to \Gamma_3, \Upsilon_2 \to \Gamma_6, \Upsilon_3 \to \Gamma_1, \Upsilon_4 \to \Gamma_5, \Upsilon_5 \to \Gamma_2, \Upsilon_6 \to \Gamma_4$.

6. Conclusion

In this paper, we have proposed two methodologies to solve the Pythagorean fuzzy assignment problem. We have solved the problem using the spherical distance measure and the exponential score function. It is anticipated that the proposed methodology is capable of managing the uncertainty persisting within the intricate assignment problem. The working of proposed technique has been illustrated via examples to test the validity. We further provide a analysis between two methodologies. Additionally, it would be engrossing to explore the application of the developed approach to picture fuzzy sets, spherical fuzzy sets and interval-valued picture fuzzy sets, etc., also to deal with other linear programming problems.

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Conflicts of Interest

The authors declare no conflicts of interest.

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