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Multi-Attribute Decision-Analytic Approach based on Spherical Fuzzy Rough Schweizer-Sklar Aggregation Operators with Applications in Agricultural Management Systems

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ABSTRACT

To find reasonable solutions for complex issues, multi-attribute group decision-making is an essential method that considers relevant attributes. For this purpose, the Schweizer–Sklar t-norms and t-conorms offer flexible and effective aggregation operators. Meanwhile, prioritized aggregation operators integrate critical information from available data to further enhance decision-making. To address uncertainty and imprecision in decision-making, in this script, we explore the spherical fuzzy rough set theory. Motivated by the utility of the Schweizer–Sklar t-norms and t-conorms, we propose a range of novel aggregation operators specifically designed for spherical fuzzy rough values, including the spherical fuzzy rough Schweizer–Sklar weighted averaging and spherical fuzzy rough Schweizer–Sklar weighted geometric operators. We examine the fundamental properties of the proposed operators in detail and demonstrate how multi-attribute group decision-making can benefit from them. A numerical example in agricultural management systems is provided to illustrate how to select the best alternative based on the given criteria. Finally, we compare the outcomes obtained using these newly postulated operators with those derived from existing studies in the literature to validate the effectiveness and practicality of the designed approaches.

1. Introduction

It frequently takes creative solutions that can handle a wide range of complex situations to introduce multi-attribute group decision-making (MAGDM) into new domains. Many approaches have been created to address these issues, especially when fuzzy data are involved. Operators with strong fuzzy data management skills have become more and more popular among these techniques.

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The history of fuzzy sets (FSs) and intuitionistic fuzzy sets (IFSs) must be traced to provide context, emphasizing both ground-breaking discoveries and more recent work. Real-world scenarios frequently involve data that is ambiguous, imprecise, and untrustworthy. To tackle these concerns, Zadeh [1] presented the idea of FSs, in which every element of uncertainty is given a membership grade (MG). This method makes the representation of uncertainty simpler, but it falls short of capturing the entire complexity of real-world situations, where the inherent ambiguity in MG determination may make it difficult to determine the non-membership grade (NMG). To overcome this restriction, Atanassov [2] introduced the idea of IFSs, which give each element an MG and NMG, with restriction $0 \leq MG + NMG \leq 1$. This development made uncertainty modeling more accurate and versatile.

On top of this base, Yager [3] presented the Pythagorean fuzzy set (PyFS), which investigates the connection between MG and NMG. Also, one more astounding commitment was made by Yager [3] in presenting the model of the q-rung orthopair fuzzy set (q-ROFS). Cuong [4] proposed the idea of picture fuzzy sets (PFS) as a solution to this problem. PFS is triplets that consist of MG, NMG, and refusal grade (RG), with the restriction that their sum cannot be greater than 1. The notions of spherical fuzzy sets (SFSs), which provide a more adaptable method of handling such constraints, were developed by Mahmood et al. [5] to overcome this drawback.

A rough set (RS) [6] makes use of the decrease and upper calculations of a crisp set to offer a green tool for coping with facts and uncertainty. RS is beneficial for reducing ambiguity and uncertainty in several disciplines. For example, Grzymala-Busse [7] investigated using RS in the paradigm of data mining. Wei *et al.* [8] checked out their software in neural networks [9].

To near the distance between FS and RS, the idea of fuzzy RS (FRS) was provided. FS is defined using decreasing and higher approximations to describe objects. Applications that include pattern choice [10], neural network choice [11], topological systems [12], and uncertainty measurement [13-14] helped to popularize this idea. In addition to this, Imran & Ullah [16] suggested the circular intuitionistic approach as a new paradigm for DM. Applications of intuitionistic FRS (IFRS) in studies encompass DM problems [19] using Frank TN and TCN [17], choosing surgical units [18-19], and resolving MAGDM.

Sahu *et al.* [20] established the concept of an RS of hybridized distance measures. Ahmed *et al.* [21] introduced the concept of an RS on the PFS and its applications. Zheng *et al.* [22] used the RS theory on the spherical fuzzy soft average aggregation operators. Hashmi *et al.* [23] developed the concepts of spherical linear on MDCM. Shen [24] introduced the a rough set-based bipolar approach for reconnoitering the relationship between financial performance indicators, ESG, and stock price returns. The Aczel-Alsina AOs to different applications were introduced in [25-27]. Kutlu Gündoğdu & Kahraman [28] explored properties and arithmetic operations of SFS. Huang *et al.* [29] developed the SFR AOs on the TOPSIS method. Mohammed *et al.* [30] used an SFR environment in the application of smart e-tourism. Sarfraz *et al.* [31] developed the concept of PAOs based on Aczel-Alsina TN and TCN.

The notions of t-norms were expanded upon by Schweizer & Sklar [32] through the creation of a family of adaptable operations called SS TN, which includes a parameter in the interval $[-\infty, 0]$. One can determine the Hamacher TN and nilpotent TN by modifying the parametric values of the SS TN. To address real-life challenges, the properties of these AOs were explored in more detail by [33]. Researchers have paid close attention to the adaptability and utility of SS TN and TCN over time. For instance, utilizing Hamy mean models and SS TN, Chen *et al.* [34] created sophisticated techniques. The theory of PAOs was applied by Garg *et al.* [35] in IF frameworks to improve DM. Khan *et al.* [36] provided fresh approaches for real-world uses in DM systems, building on the SS TN operations.

Numerous benefits are provided by the research and mathematical AOs that are discussed. Still, several experts might run into problems when compiling ambiguous data and DM. SS TN and TCN are very useful in producing accurate and trustworthy aggregated data to address these problems. We have devised a novel method that integrates TCN and SS TN with SF information to address the shortcomings and restrictions of current approaches. An important part of this work is investigating the robust features of SF theory. To aggregate unclear human opinions, we also used the operational laws of SS TN and TCN. The SF framework was extended to include several mathematical techniques, including the Schweizer–Sklar weighted averaging (SFRSSWA) and spherical fuzzy rough Schweizer–Sklar weighted geometric (SFRSSWG) operators. These techniques can determine which option is best, extract it from situations that appear unclear, and produce rankings without requiring predetermined weight data. In addition, we presented a fuzzy information decision algorithm and used it in a numerical example of agricultural management systems to find the best choice based on the developed mathematical techniques. Finally, a thorough comparative study was carried out to compare the combined results of the suggested strategies with those of the current AOs.

The rest of the article is organized as follows. For a better understanding, Section 2 provides an overview of the article's fundamental ideas. In Section 3, the notions of the SFRSSWA and SFRSSWG operators are described in detail, and their basic characteristics are examined. In Section 4, the developed AOs are applied to a MAGDM problem. First, the methodology for using the SFRSSWA and SFRSSWG operators is discussed, and then a real-world agricultural management system MAGDM problem is explained with the support of these operators. Section 5 focused on an in-depth sensitivity analysis regarding the SS parameter to check the stability of ranking outcomes. In Section 6, we execute a comparative study of the developed strategy with several existing approaches. Lastly, an overview of the article is given in Section 7. A procedural flowchart that represents the article's organization is exhibited in Figure 1.

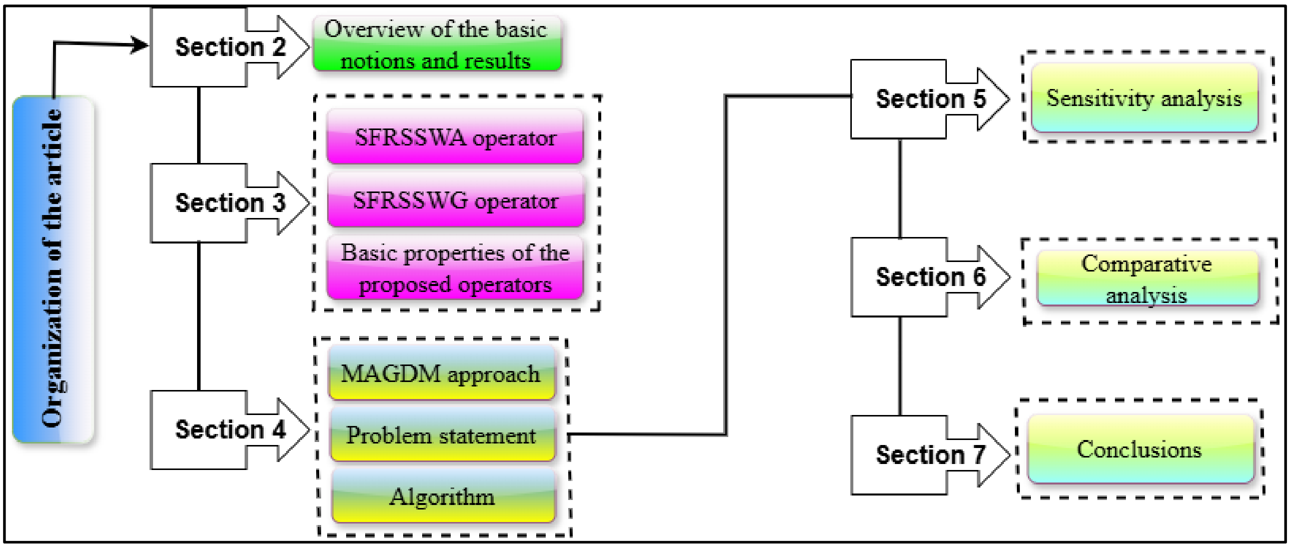

Fig. 1. Procedural flow of the proposed study

2. Preliminaries

This section includes the definitions of the primary terminologies. Some fundamental binary members of the family, SFS, SFRS, SFRV, SS TN, and SS TCN, are defined in this section.

Definition 1: [5] The SFS γ on the universe ρ is described as:

$$\boldsymbol{\gamma} = \{\boldsymbol{\xi}, (\boldsymbol{\psi}_{\xi}, \boldsymbol{\lambda}_{\xi}, \boldsymbol{\delta}_{\xi}) : \boldsymbol{\xi} \in \boldsymbol{\rho}\}, \tag{1}$$

where $\boldsymbol{\lambda}_{\xi}, \psi_{\xi}, \delta_{\xi} : \rho \longrightarrow [0,1]$, represents MG, neutral grade, and NMG such that $0 \leq \psi_{\xi}^2 + \boldsymbol{\lambda}_{\xi}^2 + \delta_{\xi}^2 \leq 1$. Consider three SFVs, $\gamma = (\psi_{\xi}, \boldsymbol{\lambda}_{\xi}, \delta_{\xi})$ and $\gamma_{\sigma} = (\psi_{\xi\sigma}, \boldsymbol{\lambda}_{\xi\sigma}, \delta_{\xi\sigma})$ for $\sigma = 1, 2$. Then, some basic properties of SFVs are given below:

i. $\boldsymbol{\gamma_1 \cup \gamma_2 = (max(\psi_{\xi 1}, \psi_{\xi 2}), min(\lambda_{\xi 1}, \lambda_{\xi 2}), min(\delta_{\xi 1}, \delta_{\xi 2}))}$.
ii. $\boldsymbol{\gamma_1 \cap \gamma_2 = (min(\psi_{\xi 1}, \psi_{\xi 2}), max(\lambda_{\xi 1}, \lambda_{\xi 2}), max(\lambda_{\xi 1}, \lambda_{\xi 2}))}$.
iii. $\boldsymbol{\gamma_1 \oplus \gamma_2 = \left(\sqrt{\psi_{\xi 1}^2 + \psi_{\xi 2}^2 - \psi_{\xi 1}^2 \psi_{\xi 2}^2}, \lambda_{\xi 1}\lambda_{\xi 2}, \delta_{\xi 1}\delta_{\xi 2}\right)}$.
iv. $\boldsymbol{\gamma_1 \otimes \gamma_2 = \left(\psi_{\xi 1}\psi_{\xi 2}, \sqrt{\lambda_{\xi 1}^2 + \lambda_{\xi 2}^2 - \lambda_{\xi 1}^2 \lambda_{\xi 2}^2} \sqrt{\delta_{\xi 1}^2 + \delta_{\xi 2}^2 - \delta_{\xi 1}^2 \delta_{\xi 2}^2}\right)}$.
v. $\boldsymbol{\gamma^c = (\lambda_{\xi}, \psi_{\xi}, \psi_{\xi})}$, where γ^c is the complement of the SFV γ.
vi. $\boldsymbol{\sigma\gamma = \left(\sqrt{1 - (1 - \psi_{\xi\rho}^2)^{\sigma}}, \lambda_{\rho}^{\sigma}, \delta_{\rho}^{\sigma}\right)}$, for $\boldsymbol{\sigma > 0}$.
vii. $\boldsymbol{\gamma^{\sigma} = \left(\psi_{\xi\rho}^{\sigma}, \sqrt{1 - (1 - \lambda_{\rho})^{2\sigma}}, \sqrt{1 - (1 - \delta_{\rho})^{2\sigma}}\right)}$, for $\boldsymbol{\sigma > 0}$.

Definition 2: Consider $\Phi \in \rho \times \rho$ to be a binary relation on the set ρ. Then, Φ is said to be:

i. Reflexive if $(M, M) \in \Phi \ \forall \ M \in \gamma$.
ii. Symmetric if $(M, \tau) = (\tau, M) \in \Phi \ \forall \ M, \tau \in \gamma$.
iii. Transitive if $(\tau, \omega) \in \Phi$ and $(\omega, M) \in \Phi$, then $(\tau, M) \in \Phi \ \forall \ M, \tau, \omega \in \gamma$.

Definition 3: [6] Let γ be the relation and the universal set Φ. Define a mapping $\Phi^*: \gamma \to \mathcal{A}(\gamma)$ given as:

$$\boldsymbol{\Phi}^*(\boldsymbol{a}) = \{\boldsymbol{M} \in \boldsymbol{\gamma} : (\boldsymbol{a}, \boldsymbol{M}) \in \boldsymbol{\Phi}\}, \text{for } \boldsymbol{a} \in \boldsymbol{\gamma}, \tag{2}$$

where $\Phi^*(a)$ is known as the successor neighborhood of a concerning Φ and (γ, Φ) is said to be the crisp approximation space. For any set $\alpha \subseteq \gamma$, definitions of the lower approximation (LA) and upper approximation (UA) are given below:

$$\boldsymbol{\Phi^{LA}(\alpha) = \{M \in \Phi | \Phi^*(M) \subseteq \alpha\}}, \tag{3}$$

$$\boldsymbol{\Phi^{UA}(\alpha) = \{M \in \Phi | \Phi^*(M) \cap \alpha \neq \phi\}}. \tag{4}$$

The set $\left(\Phi^{\text{LA}}(\alpha), \Phi^{\text{UA}}(\alpha)\right)$ is regarded as an RS w.r.t. LA and UA.

Definition 4: [6] Consider γ to be the universal set and Φ to be the relation from $SFS(\gamma \times \gamma)$. Then:

i. Φ is called reflexive if $\psi_{\Phi}(M, M) = 1$ and $\boldsymbol{\lambda}_{\Phi}(M, M) = 0 \ \forall \ M \in \gamma$.
ii. Φ is named symmetric if $\forall (M, \tau) \in \gamma \times \gamma$, then $\psi_{\Phi}(\tau, M) = \psi_{\Phi}(M, \tau) \ \forall \ M, \tau \in \gamma$ and $\boldsymbol{\lambda}_{\Phi}(\tau, M) = \boldsymbol{\lambda}_{\Phi}(M, \tau)$.
iii. Φ is termed as transitive if $\forall \ M, \tau, \omega \in \gamma$ when $(\tau, \omega) \in \Phi$ and $(\omega, M) \in \Phi$, then $\psi_{\Phi}(\tau, M) \geq \vee[\psi_{\Phi}(\tau, \omega) \wedge \psi_{\Phi}(\omega, M)]$ and $\boldsymbol{\lambda}_{\Phi}(\tau, M) \geq \wedge[\boldsymbol{\lambda}_{\Phi}(\tau, \omega) \wedge \boldsymbol{\lambda}_{\Phi}(\omega, M)]$.

Definition 5: If we take γ to be the universal set and $\Phi \in \gamma \times \gamma$ to be a spherical fuzzy relation, then the spherical fuzzy approximation space is an object of the form (γ, Φ). The characterizations of the spherical fuzzy lower approximation (SFLA) and spherical fuzzy upper approximation (SFUA) are postulated below for any set $\alpha \subseteq \gamma$:

$$\boldsymbol{\phi^{SFUA}(\alpha)} = \{\boldsymbol{M}, \boldsymbol{\psi_{\Phi^{SFUA}(\alpha)}(M)}, \boldsymbol{\lambda_{\Phi^{SFUA}(\alpha)}(M)}, \boldsymbol{\delta_{\Phi^{SFUA}(\alpha)}(M)} | \boldsymbol{M} \in \boldsymbol{\gamma}\}, \tag{5}$$

$$\boldsymbol{\Phi^{SFLA}(\alpha)} = \{\boldsymbol{M}, \boldsymbol{\psi_{\Phi^{SFLA}(\alpha)}(M)}, \boldsymbol{\lambda_{\Phi^{SFLA}(\alpha)}(M)}, \boldsymbol{\delta_{\Phi^{SFLA}(\alpha)}(M)} | \boldsymbol{M} \in \boldsymbol{\gamma}\}, \tag{6}$$

where:

$$\boldsymbol{\psi_{\Phi^{SFUA}(\alpha)}(M)} = \bigvee_{\iota \in \gamma} [\boldsymbol{\psi_{\Phi(M)}(M, \iota)} \vee \boldsymbol{\psi_{\alpha(M)}}], \tag{7}$$

$$\boldsymbol{\lambda_{\Phi^{SFUA}(\alpha)}(M)} = \bigwedge_{\iota \in \gamma} [\boldsymbol{\lambda_{\Phi(M)}(M, \iota)} \wedge \boldsymbol{\lambda_{\alpha(M)}}], \tag{8}$$

$$\boldsymbol{\delta_{\Phi^{SFUA}(\alpha)}(M)} = \bigwedge_{\iota \in \gamma} [\boldsymbol{\delta_{\Phi(M)}(M, \iota)} \wedge \boldsymbol{\delta_{\alpha(M)}}], \tag{9}$$

$$\boldsymbol{\psi_{\Phi^{SFLA}(\alpha)}(M)} = \bigwedge_{\iota \in \gamma} [\boldsymbol{\psi_{\Phi(M)}(M, \iota)} \wedge \boldsymbol{\psi_{\alpha(M)}}], \tag{10}$$

$$\boldsymbol{\lambda_{\Phi^{SFLA}(\alpha)}(M)} = \bigvee_{\iota \in \gamma} [\boldsymbol{\lambda_{\Phi(M)}(M, \iota)} \vee \boldsymbol{\lambda_{\alpha(M)}}], \tag{11}$$

$$\boldsymbol{\delta_{\Phi^{SFLA}(\alpha)}(M)} = \vee_{\iota \in \gamma} [\boldsymbol{\delta_{\Phi(M)}(M, \iota)} \vee \boldsymbol{\delta_{\alpha(M)}}], \tag{12}$$

with condition $0 \leq \psi^2_{\Phi^{\mathrm{SFUA}}(\alpha)}(M) + \boldsymbol{\lambda}^2_{\Phi^{\mathrm{SFUA}}(\alpha)}(M) + \delta^2_{\Phi^{\mathrm{SFUA}}(\alpha)}(M) \leq 1$ and $0 \leq \psi^2_{\Phi^{\mathrm{SFLA}}(\alpha)}(M) + \boldsymbol{\lambda}^2_{\Phi^{\mathrm{SFLA}}(\alpha)}(M) + \delta^2_{\Phi^{\mathrm{SFLA}}(\alpha)}(M) \leq 1$. Moreover, the set $\left(\Phi^{\mathrm{SFLA}}(\alpha), \Phi^{\mathrm{SFUA}}(\alpha)\right)$ is known as an SFRS built on SFLA and SFUA. For simplicity, $\left(\left(\underbrace{\psi^{\iota}_{\sigma}}, \underbrace{\boldsymbol{\lambda}^{\iota}_{\sigma}}, \underbrace{\delta^{\iota}_{\sigma}}\right), \left(\overbrace{\psi^{\mu}_{\sigma}}, \overbrace{\boldsymbol{\lambda}^{\mu}_{\sigma}}, \overbrace{\delta^{\mu}_{\sigma}}\right)\right)$ is named a spherical fuzzy rough value (SFRV).

Definition 6: Consider that $\sigma_1 = \left(\ell_{\sigma_1}, \wp_{\sigma_1}, \varphi_{\sigma_1}\right)$ be an SFRV in F, then score and accuracy functions for σ_1 are respectively characterized as follows:

$$\boldsymbol{Sco(\sigma_1)} = \frac{\left(\mathbf{1} + \left(\underbrace{\psi^{\iota 2}_{\sigma_1}} - \overbrace{\psi^{\mu 2}_{\sigma_2}}\right) + \left(\underbrace{\lambda^{\iota 2}_{\sigma_1}} - \overbrace{\lambda^{\mu 2}_{\sigma_2}}\right) + \left(\underbrace{\delta^{\iota 2}_{\sigma_1}} - \overbrace{\delta^{\mu 2}_{\sigma_2}}\right)\right)}{\mathbf{3}}, \tag{13}$$

$$\boldsymbol{Acc\ (\sigma_1)} = \frac{\left(\mathbf{1} + \left(\underbrace{\psi^{\iota 2}_{\sigma_1}} + \overbrace{\psi^{\mu 2}_{\sigma_2}}\right) + \left(\underbrace{\lambda^{\iota 2}_{\sigma_1}} + \overbrace{\lambda^{\mu 2}_{\sigma_2}}\right) + \left(\underbrace{\delta^{\iota 2}_{\sigma_1}} + \overbrace{\delta^{\mu 2}_{\sigma_2}}\right)\right)}{\mathbf{3}}, \tag{14}$$

where $Sco(\sigma_1) \in [-1, 1]$ and $Acc(\sigma_1) \in [0, 1]$.

To determine the order relation of two SFRVs $\sigma_1 = (\ell_{\sigma_1}, \wp_{\sigma_1}, \varphi_{\sigma_1})$ and $\psi_2 = (\ell_{\sigma_2}, \wp_{\sigma_2}, \varphi_{\sigma_2})$, the subsequent results can be utilized:

i. If $Sco(\sigma_1) < Sco(\sigma_2)$, then $\sigma_1 < \sigma_2$.
ii. If $Sco(\sigma_1) \succcurlyeq Sco(\sigma_2)$, then $\sigma_1 \succcurlyeq \sigma_2$.
iii. If $Sco(\sigma_1) = Sco(\sigma_2)$, then: if $Acc\ (\sigma_1) < Acc\ (\sigma_2)$, then $\sigma_1 < \sigma_2$; if $Acc\ (\sigma_1) \succcurlyeq Acc\ (\sigma_2)$, then $\sigma_1 \succcurlyeq \sigma_2$; and if $Acc\ (\sigma_1) = Acc\ (\sigma_2)$, then $\sigma_1 = \sigma_2$.

Definition 7: The SS TN and SS TCN are articulated as:

$$\boldsymbol{\Psi}_{SS}\,(\boldsymbol{o}, \boldsymbol{\varsigma}) = \left(\boldsymbol{o}^{\Delta} + \boldsymbol{\varsigma}^{\Delta} - \mathbf{1}\right)^{1/\Delta}, \tag{15}$$

$$\boldsymbol{\Psi}^{*}{}_{SS}(\boldsymbol{o}, \boldsymbol{\varsigma}) = \mathbf{1} - \left((\mathbf{1} - \boldsymbol{o})^{\Delta} + (\mathbf{1} - \boldsymbol{\varsigma})^{\Delta} - \mathbf{1}\right)^{1/\Delta}, \tag{16}$$

where $\Delta < 0$ and $o, \varsigma \in [0, 1]$.

3. Spherical Fuzzy Rough Schweizer-Sklar Aggregation Operators

The SS TN and SS TCN are used in this section to create a family of weighted averaging and geometric AOs. The SS TN and SS TCN are the primary foundation for defining a few operational laws for SFRVs to extend those AOs. Listed below are some basic operational legal guidelines for SFRVs.

Definition 8: Let $\mathrm{R}_\sigma = \left(\left(\underbrace{\psi}_{\sigma}^{\iota}, \underbrace{\lambda}_{\sigma}^{\iota}, \underbrace{\delta}_{\sigma}^{\iota}\right), \left(\overbrace{\psi}_{\sigma}^{\mu}, \overbrace{\lambda}_{\sigma}^{\mu}, \overbrace{\delta}_{\sigma}^{\mu}\right)\right), \sigma = 1, 2$ be two SFRVs. Then:

$$\mathrm{R}_1 \oplus \mathrm{R}_2 = \left(\begin{pmatrix} \sqrt{\mathbf{1} - \left(\left(\mathbf{1} - \underbrace{\psi}_{1}^{\iota 2}\right)^{\Delta} + \left(\mathbf{1} - \underbrace{\psi}_{2}^{\iota 2}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta}}, \\ \left(\left(\underbrace{\lambda}_{1}^{\iota}\right)^{\Delta} + \left(\underbrace{\lambda}_{2}^{\iota}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta}, \\ \left(\left(\underbrace{\delta}_{1}^{\iota}\right)^{\Delta} + \left(\underbrace{\delta}_{2}^{\iota}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta} \end{pmatrix}, \begin{pmatrix} \sqrt{\mathbf{1} - \left(\left(\mathbf{1} - \overbrace{\psi}_{1}^{\mu 2}\right)^{\Delta} + \left(\mathbf{1} - \overbrace{\psi}_{2}^{\mu 2}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta}}, \\ \left(\left(\overbrace{\lambda}_{1}^{\mu}\right)^{\Delta} + \left(\overbrace{\lambda}_{2}^{\mu}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta}, \\ \left(\left(\overbrace{\delta}_{1}^{\mu}\right)^{\Delta} + \left(\overbrace{\delta}_{2}^{\mu}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta} \end{pmatrix}\right), \tag{17}$$

$$\mathrm{R}_1 \otimes \mathrm{R}_2 = \left(\begin{pmatrix} \left(\left(\underbrace{\psi}_{1}^{\iota}\right)^{\Delta} + \left(\underbrace{\psi}_{2}^{\iota}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta}, \\ \sqrt{\mathbf{1} - \left(\left(\mathbf{1} - \underbrace{\lambda}_{1}^{\iota 2}\right)^{\Delta} + \left(\mathbf{1} - \underbrace{\lambda}_{2}^{\iota 2}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta}}, \\ \sqrt{\mathbf{1} - \left(\left(\mathbf{1} - \underbrace{\delta}_{1}^{\iota 2}\right)^{\Delta} + \left(\mathbf{1} - \underbrace{\delta}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta}} \end{pmatrix}, \begin{pmatrix} \left(\left(\overbrace{\psi}_{1}^{\mu}\right)^{\Delta} + \left(\overbrace{\psi}_{2}^{\mu}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta}, \\ \sqrt{\mathbf{1} - \left(\left(\mathbf{1} - \overbrace{\lambda}_{1}^{\mu 2}\right)^{\Delta} + \left(\mathbf{1} - \overbrace{\lambda}_{2}^{\mu 2}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta}}, \\ \sqrt{\mathbf{1} - \left(\left(\mathbf{1} - \overbrace{\delta}_{1}^{\mu 2}\right)^{\Delta} + \left(\mathbf{1} - \overbrace{\delta}_{2}^{\mu 2}\right)^{\Delta} - \mathbf{1}\right)^{1/\Delta}} \end{pmatrix}\right), \tag{18}$$

$$\boldsymbol{\eta}\mathrm{R}_1 = \left(\begin{array}{c}\left(\sqrt{1-\left(\boldsymbol{\eta}\left(1-\underbrace{\psi}_1^{\iota 2}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}}, \left(\boldsymbol{\eta}\left(\underbrace{\lambda}_1^{\iota}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}, \left(\boldsymbol{\eta}\left(\underbrace{\delta}_1^{\iota}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}\right), \\ \left(\sqrt{1-\left(\boldsymbol{\eta}\left(1-\widetilde{\psi}_1^{\mu 2}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}}, \left(\boldsymbol{\eta}\left(\widetilde{\lambda}_1^{\mu}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}, \left(\boldsymbol{\eta}\left(\widetilde{\delta}_1^{\mu}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}\right)\end{array}\right), \tag{19}$$

$$\mathrm{R}_1^{\boldsymbol{\eta}} = \left(\begin{array}{c}\left(\left(\boldsymbol{\eta}\left(\underbrace{\psi}_1^{\iota}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}, \sqrt{1-\left(\boldsymbol{\eta}\left(1-\underbrace{\lambda}_1^{\iota 2}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}}, \sqrt{1-\left(\boldsymbol{\eta}\left(1-\underbrace{\delta}_1^{\iota 2}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}}\right) \\ \left(\boldsymbol{\eta}\left(\widetilde{\psi}_1^{\mu}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}, \sqrt{1-\left(\boldsymbol{\eta}\left(1-\widetilde{\lambda}_1^{\mu 2}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}}, \sqrt{1-\left(\boldsymbol{\eta}\left(1-\widetilde{\delta}_1^{\mu 2}\right)^{\Delta}-(\boldsymbol{\eta}-1)\right)^{1/\Delta}}\end{array}\right). \tag{20}$$

Now, based on the above-mentioned operational rules, in the following, we constructed two types of AOs for SFRVs, including SFRSSWA and SFRSSWG operators.

Definition 9: Let $\mathrm{R}_\sigma = \left(\left(\underbrace{\psi}_\sigma^{\iota}, \underbrace{\lambda}_\sigma^{\iota}, \underbrace{\delta}_\sigma^{\iota}\right), \left(\widetilde{\psi}_\sigma^{\mu}, \widetilde{\lambda}_\sigma^{\mu}, \widetilde{\delta}_\sigma^{\mu}\right)\right) (\sigma = 1, 2, \cdots, \kappa)$ be an array of κ SFRVs and α_σ is the weight of the σth SFRV such that $\sum_{\sigma=1}^{\kappa}\alpha_\sigma = 1$. Then, the SFRSSWA operator is described as follows:

$$\boldsymbol{SFRSSWA}(\mathrm{R}_1, \mathrm{R}_2, \cdots, \mathrm{R}_{\boldsymbol{\kappa}}) = \bigoplus_{\sigma=1}^{\kappa} \boldsymbol{\alpha}_{\sigma}\mathrm{R}_{\sigma} =$$
$$\left(\begin{array}{c}\left(\sqrt{1-\left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(1-\underbrace{\psi}_\sigma^{\iota 2}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(\underbrace{\lambda}_\sigma^{\iota}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(\underbrace{\delta}_\sigma^{\iota}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\right) \\ \left(\sqrt{1-\left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(1-\widetilde{\psi}_\sigma^{\mu 2}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(\widetilde{\lambda}_\sigma^{\mu}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(\widetilde{\delta}_\sigma^{\mu}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\right)\end{array}\right). \tag{21}$$

Theorem 1: Consider $\mathrm{R}_\sigma = \left(\left(\underbrace{\psi}_\sigma^{\iota}, \underbrace{\lambda}_\sigma^{\iota}, \underbrace{\delta}_\sigma^{\iota}\right), \left(\widetilde{\psi}_\sigma^{\mu}, \widetilde{\lambda}_\sigma^{\mu}, \widetilde{\delta}_\sigma^{\mu}\right)\right), \sigma = 1, 2, \cdots, \kappa$ are κ SFRVs. Then, the gathered result obtained using the SFRSSWA operator is still an SFRV and is specified as follows:

$$\boldsymbol{SFRSSWA}(\mathrm{R}_1, \mathrm{R}_2, \cdots, \mathrm{R}_{\boldsymbol{\kappa}}) =$$
$$\left(\begin{array}{c}\left(\sqrt{1-\left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(1-\underbrace{\psi}_\sigma^{\iota 2}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(\underbrace{\lambda}_\sigma^{\iota}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(\underbrace{\delta}_\sigma^{\iota}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\right) \\ \left(\sqrt{1-\left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(1-\widetilde{\psi}_\sigma^{\mu 2}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(\widetilde{\lambda}_\sigma^{\mu}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\kappa}\boldsymbol{\alpha}_\sigma\left(\widetilde{\delta}_\sigma^{\mu}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\right)\end{array}\right). \tag{22}$$

Proof of Theorem 1 is provided Appendix-1.

Theorem 2 (idempotency): Consider $\mathrm{R}_\sigma = \left(\left(\underbrace{\psi^{\iota}_{\sigma}}, \underbrace{\boldsymbol{\lambda}^{\iota}_{\sigma}}, \underbrace{\delta^{\iota}_{\sigma}}\right), \left(\overbrace{\psi^{\mu}_{\sigma}}, \overbrace{\boldsymbol{\lambda}^{\mu}_{\sigma}}, \overbrace{\delta^{\mu}_{\sigma}}\right)\right), \sigma = 1, 2, \cdots, \kappa$ are κ SFRVs and $\mathrm{R}_\sigma = \left((\psi^{\iota 2}_{\sigma}, \boldsymbol{\lambda}^{\iota 2}_{\sigma}, \delta^{\iota 2}_{\sigma}), \left(\psi^{\mu 2}_{\sigma}, \boldsymbol{\lambda}^{\mu 2}_{\sigma}, \delta^{\mu 2}_{\sigma}\right)\right) = \left((\psi^{\iota 2}, \boldsymbol{\lambda}^{\iota 2}, \delta^{\iota 2}), (\psi^{\mu 2}, \boldsymbol{\lambda}^{\mu 2}, \delta^{\mu 2})\right) = \mathrm{R}, \forall \sigma = 1, 2, \cdots, \kappa$. Then:

$$\boldsymbol{SFRSSWA}(\mathrm{R}_1, \mathrm{R}_2, \cdots, \mathrm{R}_\kappa) = \left((\boldsymbol{\psi}^{\iota 2}, \boldsymbol{\lambda}^{\iota 2}, \boldsymbol{\delta}^{\iota 2}), (\boldsymbol{\psi}^{\mu 2}, \boldsymbol{\lambda}^{\mu 2}, \boldsymbol{\delta}^{\mu 2})\right) = \mathrm{R}. \tag{23}$$

Proof of Theorem 2 is provided Appendix-2.

Theorem 3 (boundedness): Consider $\mathrm{R}_\sigma = \left(\left(\underbrace{\psi^{\iota}_{\sigma}}, \underbrace{\boldsymbol{\lambda}^{\iota}_{\sigma}}, \underbrace{\delta^{\iota}_{\sigma}}\right), \left(\overbrace{\psi^{\mu}_{\sigma}}, \overbrace{\boldsymbol{\lambda}^{\mu}_{\sigma}}, \overbrace{\delta^{\mu}_{\sigma}}\right)\right), \sigma = 1, 2, \cdots, \kappa$ are κ SFRVs and R^{s}_{σ} and R^{g}_{σ} are the smallest and the greatest SFRV, respectively. Then:

$$\mathrm{R}^{s}_{\sigma} \leq \boldsymbol{SFRSSWA}(\mathrm{R}_1, \mathrm{R}_2, \cdots, \mathrm{R}_\kappa) \leq \mathrm{R}^{g}_{\sigma}. \tag{24}$$

Theorem 4 (monotonicity): Let $\mathrm{R}_\sigma = \left(\left(\underbrace{\psi^{\iota}_{\sigma}}, \underbrace{\boldsymbol{\lambda}^{\iota}_{\sigma}}, \underbrace{\delta^{\iota}_{\sigma}}\right), \left(\overbrace{\psi^{\mu}_{\sigma}}, \overbrace{\boldsymbol{\lambda}^{\mu}_{\sigma}}, \overbrace{\delta^{\mu}_{\sigma}}\right)\right)$ and $\mathrm{R}^{v}_{\sigma} = \left((\psi^{v\iota 2}_{\sigma}, \boldsymbol{\lambda}^{v\iota 2}_{\sigma}), \left(\psi^{v\mu 2}_{\sigma}, \boldsymbol{\lambda}^{v\mu 2}_{\sigma}\right)\right), \sigma = 1, 2, \cdots, \kappa$ be two sets of SFRVs such that $\mathrm{R}_\sigma \leq \mathrm{R}^{v}_{\sigma}$. Then:

$$\boldsymbol{SFRSSWA}(\mathrm{R}_1, \mathrm{R}_2, \cdots, \mathrm{R}_\kappa) \leq \boldsymbol{SFRSSWA}(\mathrm{R}^{v}_1, \mathrm{R}^{v}_2, \cdots, \mathrm{R}^{v}_\kappa). \tag{25}$$

Definition 10: Consider $\mathrm{R}_\sigma = \left(\left(\underbrace{\psi^{\iota}_{\sigma}}, \underbrace{\lambda^{\iota}_{\sigma}}, \underbrace{\delta^{\iota}_{\sigma}}\right), \left(\overbrace{\psi^{\mu}_{\sigma}}, \overbrace{\boldsymbol{\lambda}^{\mu}_{\sigma}}, \overbrace{\delta^{\mu}_{\sigma}}\right)\right), \sigma = 1, 2, \cdots, \kappa$ are κ SFRVs and α_σ denotes the weight of the σth SFRV. Then, the SFRSSWAG operator is postulated as:

$$\boldsymbol{SFRAAWG}(\mathrm{R}_1, \mathrm{R}_2, \cdots, \mathrm{R}_\kappa) = \bigotimes_{\sigma=1}^{\kappa} \mathrm{R}^{\alpha_\sigma}_{\sigma} =$$
$$\left(\begin{array}{c}\left(\left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(\underbrace{\boldsymbol{\psi}^{\iota}_{\sigma}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \sqrt{1 - \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(1 - \underbrace{\boldsymbol{\lambda}^{\iota 2}_{\sigma}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \sqrt{1 - \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(1 - \underbrace{\boldsymbol{\delta}^{\iota 2}_{\sigma}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}\right), \\ \left(\left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(\overbrace{\boldsymbol{\psi}^{\mu}_{\sigma}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \sqrt{1 - \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(1 - \overbrace{\boldsymbol{\lambda}^{\mu 2}_{\sigma}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \sqrt{1 - \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(1 - \overbrace{\boldsymbol{\delta}^{\mu 2}_{\sigma}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}\right)\end{array}\right). \tag{26}$$

Theorem 5: Consider $\mathrm{R}_\sigma = \left(\left(\underbrace{\psi^{\iota}_{\sigma}}, \underbrace{\boldsymbol{\lambda}^{\iota}_{\sigma}}, \underbrace{\delta^{\iota}_{\sigma}}\right), \left(\overbrace{\psi^{\mu}_{\sigma}}, \overbrace{\boldsymbol{\lambda}^{\mu}_{\sigma}}, \overbrace{\delta^{\mu}_{\sigma}}\right)\right), \sigma = 1, 2, \cdots, \kappa$ are κ SFRVs. Then, SFRV is obtained after the aggregation from the SFRSSWG operator is again SFRV and is described as:

$$\boldsymbol{SFRAAWG}(\mathrm{R}_{\mathbf{1}},\mathrm{R}_{\mathbf{2}},\cdots,\mathrm{R}_{\boldsymbol{\kappa}})=$$
$$\left(\begin{array}{c}\left(\left(\sum_{\sigma=1}^{\kappa}\alpha_{\sigma}\left(\underbrace{\psi_{\sigma}^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}},\sqrt{1-\left(\sum_{\sigma=1}^{\kappa}\alpha_{\sigma}\left(1-\underbrace{\lambda_{\sigma}^{\iota2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}},\sqrt{1-\left(\sum_{\sigma=1}^{\kappa}\alpha_{\sigma}\left(1-\underbrace{\delta_{\sigma}^{\iota2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}\right)\\ \left(\left(\sum_{\sigma=1}^{\kappa}\alpha_{\sigma}\left(\overbrace{\psi_{\sigma}^{\mu}}\right)^{\Delta}\right)^{\frac{1}{\Delta}},\sqrt{1-\left(\sum_{\sigma=1}^{\kappa}\alpha_{\sigma}\left(1-\overbrace{\lambda_{\sigma}^{\mu2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}},\sqrt{1-\left(\sum_{\sigma=1}^{\kappa}\alpha_{\sigma}\left(1-\overbrace{\delta_{\sigma}^{\mu2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}\right)\end{array}\right). \quad (27)$$

Proof: Similar to proof of Theorem 1.

Theorem 6 (idempotency): Consider $\mathrm{R}_{\sigma}=\left(\left(\underbrace{\psi_{\sigma}^{\iota}},\underbrace{\lambda_{\sigma}^{\iota}},\underbrace{\delta_{\sigma}^{\iota}}\right),\left(\overbrace{\psi_{\sigma}^{\mu}},\overbrace{\lambda_{\sigma}^{\mu}},\overbrace{\delta_{\sigma}^{\mu}}\right)\right),\sigma=1,2,\cdots,\kappa$ are κ SFRVs and $\mathrm{R}_{\sigma}=\left(\left(\underbrace{\psi_{\sigma}^{\iota}},\underbrace{\lambda_{\sigma}^{\iota}},\underbrace{\delta_{\sigma}^{\iota}}\right),\left(\overbrace{\psi_{\sigma}^{\mu}},\overbrace{\lambda_{\sigma}^{\mu}},\overbrace{\delta_{\sigma}^{\mu}}\right)\right)=\left((\psi^{\iota2},\lambda^{\iota2},\delta^{\iota2}),(\psi^{\mu2},\lambda^{\mu2},\delta^{\mu2})\right)=\mathrm{R},\forall\sigma=1,2,\cdots,\kappa$. Then:

$$\boldsymbol{SFRSSWG}(\mathrm{R}_{\mathbf{1}},\mathrm{R}_{\mathbf{2}},\cdots,\mathrm{R}_{\boldsymbol{\kappa}})=\left((\boldsymbol{\psi}^{\iota2},\boldsymbol{\lambda}^{\iota2},\boldsymbol{\delta}^{\iota2}),(\boldsymbol{\psi}^{\mu2},\boldsymbol{\lambda}^{\mu2},\boldsymbol{\delta}^{\mu2})\right)=\mathrm{R}. \quad (28)$$

Proof: Similar to proof of Theorem 2.

Theorem 7 (boundedness): Consider $\mathrm{R}_{\sigma}=\left(\left(\underbrace{\psi_{\sigma}^{\iota}},\underbrace{\lambda_{\sigma}^{\iota}},\underbrace{\delta_{\sigma}^{\iota}}\right),\left(\overbrace{\psi_{\sigma}^{\mu}},\overbrace{\lambda_{\sigma}^{\mu}},\overbrace{\delta_{\sigma}^{\mu}}\right)\right),\sigma=1,2,\cdots,\kappa$ are κ SFRVs and R_{σ}^{s} and R_{σ}^{g} are the smallest and largest SFRVs, respectively. Then:

$$\mathrm{R}_{\boldsymbol{\sigma}}^{\boldsymbol{s}}\leq\boldsymbol{SFRSSWG}(\mathrm{R}_{\mathbf{1}},\mathrm{R}_{\mathbf{2}},\cdots,\mathrm{R}_{\boldsymbol{\kappa}})\leq\mathrm{R}_{\boldsymbol{\sigma}}^{\boldsymbol{g}}. \quad (29)$$

Proof: Analogous to proof of Theorem 3.

Theorem 8 (monotonicity): Let $\mathrm{R}_{\sigma}=\left(\left(\underbrace{\psi_{\sigma}^{\iota}},\underbrace{\lambda_{\sigma}^{\iota}},\underbrace{\delta_{\sigma}^{\iota}}\right),\left(\overbrace{\psi_{\sigma}^{\mu}},\overbrace{\lambda_{\sigma}^{\mu}},\overbrace{\delta_{\sigma}^{\mu}}\right)\right)$ and $\mathrm{R}_{\sigma}^{v}=\left(\left(\underbrace{\psi_{\sigma}^{v\iota}},\underbrace{\lambda_{\sigma}^{v\iota}},\underbrace{\delta_{\sigma}^{v\iota}}\right),\left(\overbrace{\psi_{\sigma}^{v\mu}},\overbrace{\lambda_{\sigma}^{v\mu}},\overbrace{\delta_{\sigma}^{v\mu}}\right)\right)$ $(\sigma=1,2,\cdots,\kappa)$ be two collections of SFRVs such that $\mathrm{R}_{\sigma}\leq\mathrm{R}_{\sigma}^{v}$. Then:

$$\boldsymbol{SFRSSWG}(\mathrm{R}_{\mathbf{1}},\mathrm{R}_{\mathbf{2}},\cdots,\mathrm{R}_{\boldsymbol{\kappa}})\leq\boldsymbol{SFRSSWG}(\mathrm{R}_{\mathbf{1}}^{\boldsymbol{v}},\mathrm{R}_{\mathbf{2}}^{\boldsymbol{v}},\cdots,\mathrm{R}_{\boldsymbol{\kappa}}^{\boldsymbol{v}}). \quad (30)$$

Proof: Identical to proof of Theorem 4.

4. MAGDM Method Based on the Suggested Operators

Based on assessments from experts in a range of attributes, the MAGDM process is essential in determining which of a set of desirable options is the most suitable. Experts evaluate all options based on shared characteristics and provide their conclusions as SFRVs. After that, the data gathered from these experts in decision matrices is combined while accounting for their weights. The weights assigned to the attributes are taken into consideration when further combining this aggregated data for each alternative. MAGDM is a useful method used in many fields, such as engineering, business, economics, and mathematics.

4.1 Algorithm

Figure 2 illustrates the graphical representation of the recommended MAGDM framework. Assume $\{\mathfrak{Y}_1, \mathfrak{Y}_2, \cdots, \mathfrak{Y}_r\}$ represents r alternatives assessed based on attributes $\{\hbar_1, \hbar_2, \cdots, \hbar_r\}$ by $\{\mathcal{Z}_1, \mathcal{Z}_2, \cdots, \mathcal{Z}_h\}$ specialists with weights $\mathcal{W}_\varphi \in [0,1]$ $(\varphi = 1, 2, \cdots, h)$ such that $\sum_{\varphi=1}^{h} \mathcal{W}_\varphi = 1$. The goal is to identify the most appropriate alternative between $\{\mathfrak{Y}_1, \mathfrak{Y}_2, \cdots, and\ \mathfrak{Y}_r\}$ using MAGDM.

The steps involved in selecting an alternative are as follows:

Step 1 – Experts provide their evaluations in the form of SFRVs as $\left(\left(\underset{\smile}{\psi}{}_{\sigma}^{\iota}, \underset{\smile}{\lambda}{}_{\sigma}^{\iota}, \underset{\smile}{\delta}{}_{\sigma}^{\iota}\right), \left(\overset{\frown}{\psi}{}_{\sigma}^{\mu}, \overset{\frown}{\lambda}{}_{\sigma}^{\mu}, \overset{\frown}{\delta}{}_{\sigma}^{\mu}\right)\right)$. Typically, around are two types of attributes. If a cost-type attribute is present, its complement is taken, known as normalization, resulting in $\left(\left(\underset{\smile}{\psi}{}_{\sigma}^{\iota}, \underset{\smile}{\lambda}{}_{\sigma}^{\iota}, \underset{\smile}{\delta}{}_{\sigma}^{\iota}\right), \left(\overset{\frown}{\psi}{}_{\sigma}^{\mu}, \overset{\frown}{\lambda}{}_{\sigma}^{\mu}, \overset{\frown}{\delta}{}_{\sigma}^{\mu}\right)\right)$.

Step 2 – The matrices' information is ready for aggregation following normalization. The attributes are aggregated one by one with the use of the SFRVSSWA/SFRSSWG operator, generating an aggregated selection matrix with SFRVs for every opportunity concerning every characteristic.

Step 3 – The attributes for every opportunity are then blended with the usage of the SFRSSWA/SFRSSWG operator. SFRVs, or collectively aggregated values, are the result of this step.

Step 4 – Each opportunity's score is decided using applying the score feature.

Step 5 – Lastly, a rating of the options is decided by assigning a score to each alternative.

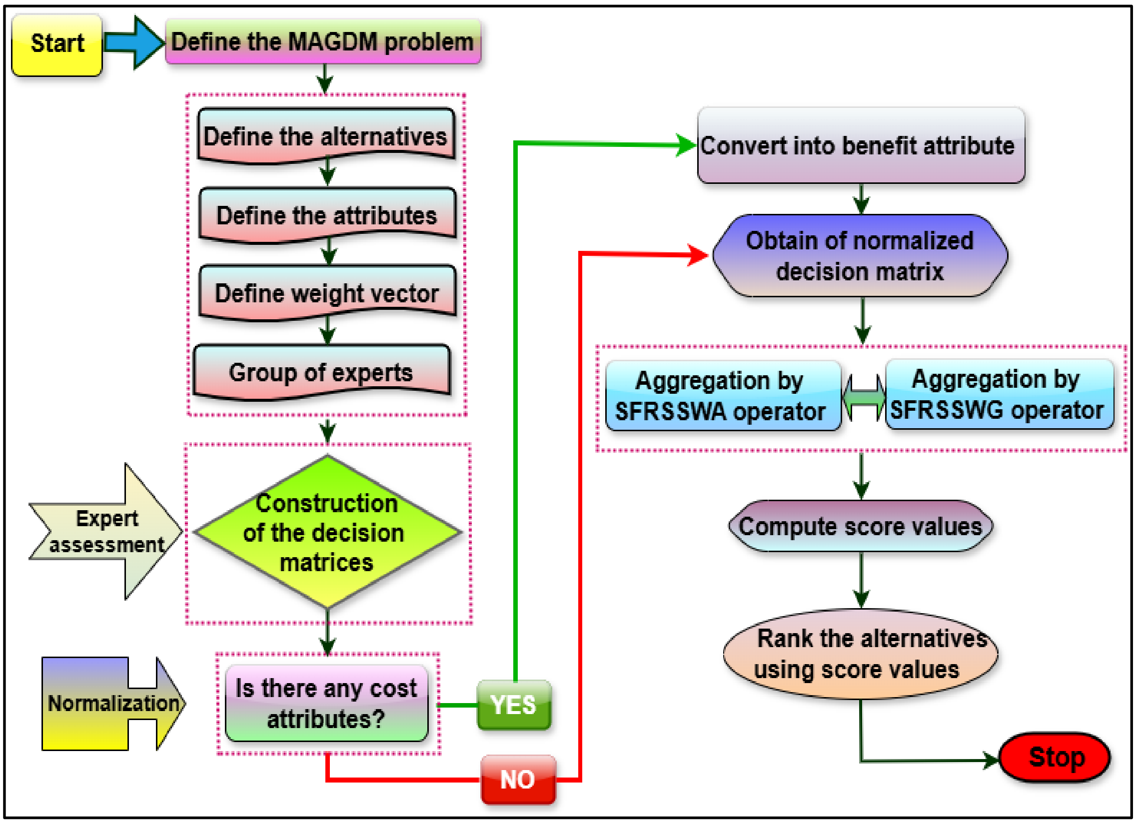

Fig. 2. Flowchart of the proposed MAGDM scheme

4.2 Numerical Example: Agriculture's Beneficial Effects on Pakistan's Economy

Agriculture is a vital aspect of Pakistan's way of life and its economic foundation. More than half of the staff are hired by way of it, and it generates 20% of the GDP of the nation, in addition to providing raw materials to many other industries. Even though Pakistan's economic system depends heavily on agriculture, the enterprise has no longer benefited from the specified investment or technological advancements. It runs the risk of lagging behind different, faster-growing industries if primary modifications are not made. This article will go over the critical function that Pakistan's agriculture enterprise performs in the welfare and monetary improvement of the country, the measures that have to be taken to enable it to realize its full potential, and its contribution to job creation and food security. Pakistan's economy has usually been, in large part, dependent on agriculture. The enterprise contributes substantially to GDP and employs a huge proportion of the labor force.

Pakistan's economy is still predicated heavily on agriculture, even as interest shifts to other areas. Some of the most pleasant vegetation in the world can be produced in this nation because of its rich soil and exceptional climate. The agricultural sector in Pakistan is sustainable and diverse, and involves fisheries, livestock, and crops. It must come as no surprise that Pakistan's authority values agriculture and strives to uphold and develop this critical enterprise. Pakistan's potential to preserve and increase agricultural output may be a first-rate factor in determining its destiny for economic achievement. Pakistan's agricultural enterprise has confronted numerous problems lately, making it more difficult for farmers to make a living. One vast contributing factor to the unpredictability of weather patterns and excessive temperatures, which have led to decreased crop yields and higher rates of pest infestations, is climate change. Lack of funding is a primary trouble as well as it prevents farmers from investing in new generations, systems, and better farming strategies. Resolving these issues is important to Pakistan's agricultural region's long-term survival and enlargement.

We evaluated several important crops in this numerical case study, including $(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4, \mathcal{F}_5)$, which are very profitable and contribute significantly to the growth of any nation's economy. When conducting this evaluation, the decision-maker takes into account several crucial traits or qualities, which are described in detail below:

i. *Supplying raw materials* (H_1) – The period "crop uncooked materials" describes plant-based assets that are taken from crops and used as the basis for plenty of goods and companies. Different plant elements, including seeds, leaves, stems, and roots, are used to make these substances. They are the building blocks of many exceptional products and business approaches. These crucial agricultural components are important for meeting purchaser needs, selling monetary benefits, and advancing the era. They are utilized by many industries, including bioenergy, textiles, agriculture, food processing, and prescribed drugs.

ii. *Economic development and the creation of jobs* (H_2) – A thriving financial system and better dwelling standards rely on the principles of activity introduction and economic introduction being intertwined. Jobs are created because of monetary development, and jobs in turn are the main driving force of economic development. Economic development and employment possibilities must always be expanding for there to be a thriving and dynamic economic system.

iii. *Constructing a sturdy supply chain* (H_3) – Building an effective and well-coordinated network of individuals, institutions, equipment, and approaches that collaborate to transfer services or products from suppliers to the very last customers is essential to establishing a robust supply chain. Sustaining market competitiveness, controlling costs, and gratifying purchaser demand all depend upon a robust delivery chain. This includes meticulous planning, constant investment, and continuous improvement to guarantee prompt and least expensive product transport, thereby augmenting a business's profitability and competitive advantage.

iv. *Industrial goods* (H_4) – Materials, chemical substances, and different materials used as inputs or raw substances in numerous business strategies are examples of business products derived from crops. These agricultural products are often processed into commodities or raw materials that are applied in industries other than agriculture.

The evaluation of a dominant crop is demonstrated using the derived methods of the SFRSSWA and SFRSSWG operators without the use of a weight vector. Furthermore, we used the SFRSSWA and RFRSSWG operators to apply weight vectors $(0.37,0.38,0.25)^T$ to determine the best crop. Within the confines of the MAGDM problem, the analysis goes on. The assessment was created on the attributes with weights $(0.21, 0.29,0.23,0.27)^T$. The experts assess each option according to the previously listed criteria. The information is given by the experts in the form of SFRVs. Table 1, Table 2, and Table 3 present the data that were acquired from expert 1, expert 2, and expert 3, respectively.

Table 1
SFRVs provided by the expert $\mathcal{Z}_1$

	$\mathfrak{S}_1$						$\mathfrak{S}_2$					
	ψ^{ul}	$\psi^{\mu l}$	λ^{ul}	$\lambda^{\mu l}$	δ^{ul}	$\delta^{\mu l}$	ψ^{ul}	$\psi^{\mu l}$	λ^{ul}	$\lambda^{\mu l}$	δ^{ul}	$\delta^{\mu l}$
$\mathcal{F}_1$	0.73	0.67	0.78	0.73	0.95	0.83	0.98	0.94	0.83	0.78	0.85	0.69
$\mathcal{F}_2$	0.75	0.74	0.81	0.79	0.93	0.81	0.83	0.82	0.82	0.79	0.83	0.85
$\mathcal{F}_3$	0.84	0.67	0.73	0.72	0.99	0.84	0.88	0.85	0.89	0.83	0.92	0.89
$\mathcal{F}_4$	0.78	0.73	0.75	0.74	0.89	0.81	0.93	0.93	0.95	0.93	0.94	0.68
	$\mathfrak{S}_3$						$\mathfrak{S}_4$					
	ψ^{ul}	$\psi^{\mu l}$	λ^{ul}	$\lambda^{\mu l}$	δ^{ul}	$\delta^{\mu l}$	ψ^{ul}	$\psi^{\mu l}$	λ^{ul}	$\lambda^{\mu l}$	δ^{ul}	$\delta^{\mu l}$
$\mathcal{F}_1$	0.78	0.68	0.95	0.93	0.93	0.89	0.89	0.78	0.98	0.95	0.98	0.93
$\mathcal{F}_2$	0.83	0.77	0.89	0.81	0.94	0.92	0.84	0.75	0.89	0.85	0.85	0.84
$\mathcal{F}_3$	0.95	0.85	0.88	0.84	0.95	0.67	0.77	0.72	0.86	0.83	0.87	0.79
$\mathcal{F}_4$	0.86	0.83	0.79	0.73	0.89	0.85	0.82	0.67	0.89	0.88	0.68	0.58

Table 2
SFRVs provided by the expert $\mathcal{Z}_2$

	$\mathfrak{S}_1$						$\mathfrak{S}_2$					
	ψ^{ul}	$\psi^{\mu l}$	λ^{ul}	$\lambda^{\mu l}$	δ^{ul}	$\delta^{\mu l}$	ψ^{ul}	$\psi^{\mu l}$	λ^{ul}	$\lambda^{\mu l}$	δ^{ul}	$\delta^{\mu l}$
$\mathcal{F}_1$	0.94	0.89	0.82	0.81	0.76	0.69	0.78	0.63	0.93	0.91	0.93	0.42
$\mathcal{F}_2$	0.84	0.79	0.89	0.85	0.87	0.85	0.79	0.66	0.96	0.95	0.91	0.34
$\mathcal{F}_3$	0.92	0.91	0.93	0.92	0.93	0.89	0.73	0.65	0.97	0.89	0.98	0.44
$\mathcal{F}_4$	0.89	0.85	0.78	0.77	0.76	0.68	0.89	0.83	0.89	0.85	0.95	0.45
	$\mathfrak{S}_3$						$\mathfrak{S}_4$					
	ψ^{ul}	$\psi^{\mu l}$	λ^{ul}	$\lambda^{\mu l}$	δ^{ul}	$\delta^{\mu l}$	ψ^{ul}	$\psi^{\mu l}$	λ^{ul}	$\lambda^{\mu l}$	δ^{ul}	$\delta^{\mu l}$
$\mathcal{F}_1$	0.77	0.56	0.68	0.66	0.84	0.69	0.68	0.59	0.87	0.85	0.79	0.63
$\mathcal{F}_2$	0.86	0.76	0.65	0.62	0.79	0.73	0.78	0.75	0.95	0.94	0.75	0.72
$\mathcal{F}_3$	0.93	0.89	0.69	0.59	0.75	0.69	0.92	0.89	0.93	0.91	0.96	0.95
$\mathcal{F}_4$	0.92	0.86	0.83	0.78	0.83	0.73	0.92	0.91	0.86	0.85	0.88	0.82

Table 3
SFRVs provided by the expert $\mathcal{Z}_3$

	$\mathfrak{S}_1$						$\mathfrak{S}_2$					
	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$
$\mathcal{F}_1$	**0.95**	**0.94**	**0.93**	**0.92**	**0.85**	**0.83**	**0.72**	**0.63**	**0.78**	**0.73**	**0.82**	**0.83**
$\mathcal{F}_2$	**0.93**	**0.88**	**0.91**	**0.89**	**0.83**	**0.67**	**0.84**	**0.73**	**0.95**	**0.93**	**0.84**	**0.81**
$\mathcal{F}_3$	**0.91**	**0.87**	**0.98**	**0.95**	**0.92**	**0.84**	**0.92**	**0.84**	**0.89**	**0.85**	**0.92**	**0.84**
$\mathcal{F}_4$	**0.92**	**0.91**	**0.95**	**0.93**	**0.94**	**0.89**	**0.85**	**0.83**	**0.79**	**0.74**	**0.89**	**0.81**
	$\mathfrak{S}_3$						$\mathfrak{S}_4$					
	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$
$\mathcal{F}_1$	**0.67**	**0.63**	**0.78**	**0.72**	**0.95**	**0.93**	**0.78**	**0.76**	**0.98**	**0.95**	**0.92**	**0.87**
$\mathcal{F}_2$	**0.83**	**0.82**	**0.85**	**0.82**	**0.89**	**0.83**	**0.89**	**0.88**	**0.87**	**0.85**	**0.85**	**0.84**
$\mathcal{F}_3$	**0.94**	**0.93**	**0.89**	**0.79**	**0.88**	**0.67**	**0.95**	**0.93**	**0.89**	**0.85**	**0.78**	**0.75**
$\mathcal{F}_4$	**0.87**	**0.85**	**0.91**	**0.86**	**0.83**	**0.68**	**0.92**	**0.91**	**0.96**	**0.95**	**0.95**	**0.94**

Step 1 & **Step 2** – To one at a time combine the attributes, the SFRSSWA and SFRSSWG operators are used. The individual aggregated values of the attributes that were acquired through using the SFRSSWA and SFRSSWG operators are displayed in Table 4 and Table 5, respectively, attributable to these aggregations.

Table 4
Aggregated values by the SFRSSWA operator

	$\mathfrak{S}_1$						$\mathfrak{S}_2$					
	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$
$\mathcal{F}_1$	**0.3974**	**0.2982**	**0.3559**	**0.3109**	**0.3802**	**0.2907**	**0.3826**	**0.2411**	**0.3754**	**0.3163**	**0.3864**	**0.8526**
$\mathcal{F}_2$	**0.3560**	**0.3072**	**0.3789**	**0.3446**	**0.3931**	**0.2990**	**0.3351**	**0.2628**	**0.4202**	**0.3629**	**0.3758**	**0.4700**
$\mathcal{F}_3$	**0.3975**	**0.2965**	**0.4029**	**0.3249**	**0.4534**	**0.3660**	**0.3625**	**0.2780**	**0.4274**	**0.3645**	**0.4466**	**0.5429**
$\mathcal{F}_4$	**0.3756**	**0.3194**	**0.3473**	**0.3037**	**0.3773**	**0.2879**	**0.4027**	**0.3645**	**0.4025**	**0.3410**	**0.4351**	**0.4897**
	$\mathfrak{S}_3$						$\mathfrak{S}_4$					
	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$
$\mathcal{F}_1$	**0.2850**	**0.1914**	**0.3576**	**0.5403**	**0.4113**	**0.3058**	**0.3262**	**0.2365**	**0.4469**	**0.3972**	**0.4150**	**0.2777**
$\mathcal{F}_2$	**0.3545**	**0.3017**	**0.3372**	**0.5173**	**0.3893**	**0.3188**	**0.3489**	**0.2983**	**0.4150**	**0.3799**	**0.3353**	**0.3062**
$\mathcal{F}_3$	**0.4419**	**0.3857**	**0.3480**	**0.4960**	**0.3848**	**0.2295**	**0.3945**	**0.3217**	**0.4020**	**0.3680**	**0.4006**	**0.3277**
$\mathcal{F}_4$	**0.3938**	**0.3574**	**0.3544**	**0.5980**	**0.3653**	**0.2797**	**0.3954**	**0.2988**	**0.4055**	**0.3837**	**0.3674**	**0.2521**

Table 5
Aggregated values by the SFRSSWG operator

	$\mathfrak{S}_1$						$\mathfrak{S}_2$					
	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$
$\mathcal{F}_1$	**0.3668**	**0.3311**	**0.3402**	**0.3346**	**0.3123**	**0.3375**	**0.3327**	**0.3149**	**0.3479**	**0.3498**	**0.2492**	**0.9911**
$\mathcal{F}_2$	**0.3229**	**0.3272**	**0.3549**	**0.3649**	**0.3233**	**0.3798**	**0.2832**	**0.3323**	**0.4049**	**0.3812**	**0.2975**	**0.8605**
$\mathcal{F}_3$	**0.3549**	**0.3852**	**0.3876**	**0.3309**	**0.3704**	**0.4387**	**0.3175**	**0.3236**	**0.3697**	**0.4109**	**0.3249**	**0.9524**
$\mathcal{F}_4$	**0.3494**	**0.3509**	**0.3353**	**0.3110**	**0.3217**	**0.3407**	**0.3818**	**0.3935**	**0.3756**	**0.3713**	**0.2398**	**0.9436**
	$\mathfrak{S}_3$						$\mathfrak{S}_4$					
	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$	$\psi^{\iota l}$	$\psi^{\mu l}$	$\lambda^{\iota l}$	$\lambda^{\mu l}$	$\delta^{\iota l}$	$\delta^{\mu l}$
$\mathcal{F}_1$	**0.1999**	**0.2762**	**0.3368**	**0.5794**	**0.3643**	**0.3893**	**0.2642**	**0.2869**	**0.4215**	**0.4118**	**0.3595**	**0.3618**
$\mathcal{F}_2$	**0.3047**	**0.3531**	**0.2934**	**0.5658**	**0.3569**	**0.3578**	**0.3148**	**0.3372**	**0.3958**	**0.4031**	**0.3235**	**0.3221**
$\mathcal{F}_3$	**0.3942**	**0.4406**	**0.2957**	**0.6061**	**0.2298**	**0.3364**	**0.3691**	**0.3508**	**0.3777**	**0.3933**	**0.3726**	**0.3631**
$\mathcal{F}_4$	**0.3586**	**0.3871**	**0.3119**	**0.6813**	**0.3026**	**0.3593**	**0.3657**	**0.3759**	**0.3966**	**0.3917**	**0.3355**	**0.3020**

Sept 3 – The next step is to use the SFRSSWA and SFRSSWG operators to aggregate these attribute values for each criterion collectively. Table 6 and Table 7 display the total aggregated values attained after these operators.

Table 6
Total values acquired by the SFRSSWA operator

$\mathfrak{S}_1$						$\mathfrak{S}_2$					
ψ^{ι}	ψ^{μ}	λ^{ι}	λ^{μ}	δ^{ι}	δ^{μ}	ψ^{ι}	ψ^{μ}	λ^{ι}	λ^{μ}	δ^{ι}	δ^{μ}
0.1836	**0.0373**	**0.1845**	**0.0404**	**0.1999**	**0.0367**	**0.1732**	**0.0339**	**0.1932**	**0.0931**	**0.1989**	**0.1252**
$\mathfrak{S}_3$						$\mathfrak{S}_4$					
ψ^{ι}	ψ^{μ}	λ^{ι}	λ^{μ}	δ^{ι}	δ^{μ}	ψ^{ι}	ψ^{μ}	λ^{ι}	λ^{μ}	δ^{ι}	δ^{μ}
0.1958	**0.0368**	**0.1676**	**0.1141**	**0.1803**	**0.0337**	**0.1821**	**0.0321**	**0.1874**	**0.0584**	**0.1846**	**0.0323**

Table 7
Total values acquired by the SFRSSWG operator

$\mathfrak{S}_1$						$\mathfrak{S}_2$					
ψ^{ι}	ψ^{μ}	λ^{ι}	λ^{μ}	δ^{ι}	δ^{μ}	ψ^{ι}	ψ^{μ}	λ^{ι}	λ^{μ}	δ^{ι}	δ^{μ}
0.1697	**0.0472**	**0.1794**	**0.0444**	**0.1737**	**0.0531**	**0.1578**	**0.0471**	**0.1752**	**0.1107**	**0.1586**	**0.3417**
$\mathfrak{S}_3$						$\mathfrak{S}_4$					
ψ^{ι}	ψ^{μ}	λ^{ι}	λ^{μ}	δ^{ι}	δ^{μ}	ψ^{ι}	ψ^{μ}	λ^{ι}	λ^{μ}	δ^{ι}	δ^{μ}
0.1809	**0.0505**	**0.1499**	**0.1429**	**0.1286**	**0.0526**	**0.1734**	**0.0450**	**0.1791**	**0.0634**	**0.1749**	**0.0445**

Step 4 – In this step, the score values for SFRSSWG and SFRSSWA were obtained. For SFRSSWA, we got $Sco(H_1) = 0.368, Sco(H_2) = 0.299, Sco(H_3) = 0.347, Sco(H_4) = 0.298$. For SFRSSWG, we calculated $Sco(H_1) = 0.361, Sco(H_2) = 0.306, Sco(H_3) = 0.343, Sco(H_4) = 0.305$. The outcomes derived from each of the projected operators are visually presented in Figure 3.

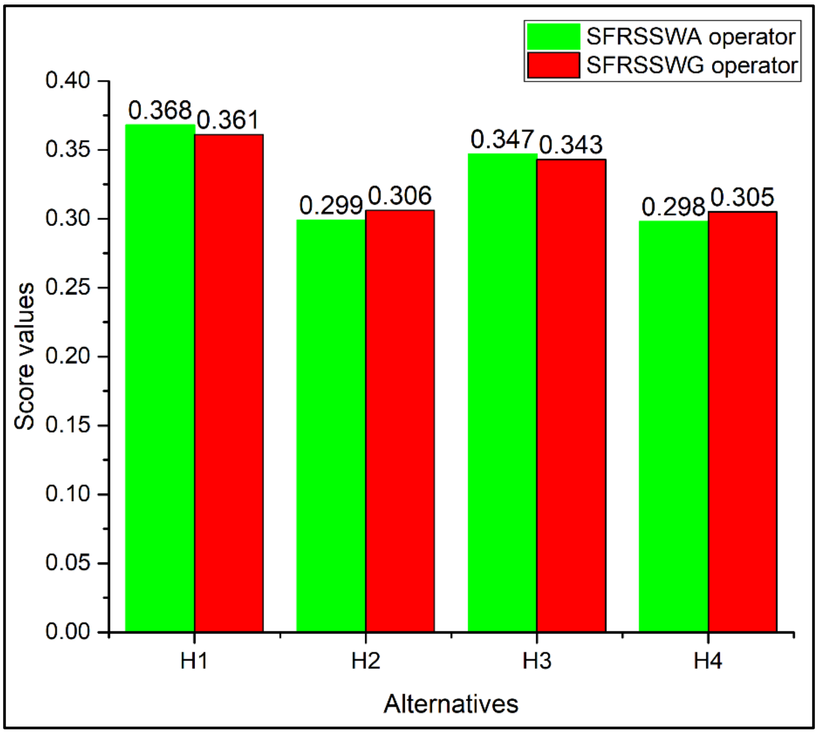

Fig. 3. Graphical depiction of the alternative factor rankings using the proposed operators

Step 5 – We ranked the agricultural alternative factors and determined which one had the greatest impact on Pakistan's economy. Table 8 presents the final ranking of alternatives.

Table 8
Ranks of the agricultural alternative factors

Operators	Ranks
SFRSSWA	$H_1 \succ H_3 \succ H_2 \succ H_4$
SFRSSWG	$H_1 \succ H_3 \succ H_2 \succ H_4$

According to Table 8, the alternative factor H_1 emerges as the most favored option under both the SFRSSWA and SFRSSWG operators. Additionally, the ranking outcomes derived from each of the projected operators are visually presented in Figure 3.

5. Sensitivity Analysis

The SFRSSWG and SFRSSWA operators incorporate the intrinsic parameter Δ, which endows them with enhanced flexibility and efficacy in data aggregation. Depending on the preferences of the decision-maker, different values can be assigned to Δ, allowing the aggregation behaviour to be tailored accordingly. To evaluate the influence of Δ on the performance of the proposed MAGDM framework, a sensitivity analysis is accomplished by considering a range of parameter values: $\Delta = -3, -5, -10, -20, -30, -50, -60, -80, -100$. This analysis is carried out using both the SFRSSWA and SFRSSWG operators. The resultant aggregated score values and ranking results of the SFRSSWA and SFRSSWG operators for the alternative factors derived from these varying Δ values are summarized in Table 9 and Table 10, respectively.

According to Table 9, we observed that as the magnitude of Δ decreases (i.e., becomes more negative), the corresponding score values for all alternatives show a monotonic increase for the alternative factors H_1 and H_3, while the scores of the alternative factors H_2 and H_4 decrease consistently. Despite the score variations, the ranking order remains consistent across all inputs of Δ. Particularly, the ranking $H_1 \succ H_3 \succ H_2 \succ H_4$ is preserved throughout the entire parameter range, demonstrating the stability of the SFRSSWA operator in DM under variable degrees of aggregation strictness.

Table 9
Sensitivity of Δ under the SFRSSWA operator

Δ	Score values				Ranking
	$Sco(H_1)$	$Sco(H_2)$	$Sco(H_3)$	$Sco(H_4)$	
-3	0.3680	0.2990	0.3470	0.2980	$H_1 \succ H_3 \succ H_2 \succ H_4$
-5	0.410	0.2570	0.3620	0.2560	$H_1 \succ H_3 \succ H_2 \succ H_4$
-10	0.4730	0.1940	0.3830	0.1930	$H_1 \succ H_3 \succ H_2 \succ H_4$
-20	0.5210	0.1450	0.3980	0.1440	$H_1 \succ H_3 \succ H_2 \succ H_4$
-30	0.540	0.1260	0.4040	0.1250	$H_1 \succ H_3 \succ H_2 \succ H_4$
-50	0.5570	0.1140	0.4090	0.110	$H_1 \succ H_3 \succ H_2 \succ H_4$
-60	0.5610	0.1060	0.410	0.1050	$H_1 \succ H_3 \succ H_2 \succ H_4$
-80	0.5670	0.1014	0.4120	0.1012	$H_1 \succ H_3 \succ H_2 \succ H_4$
-100	0.570	0.0990	0.4130	0.0970	$H_1 \succ H_3 \succ H_2 \succ H_4$

The effect of Δ upon the ranking behavior under the SFRSSWA operator is graphically demonstrated in Figure 4.

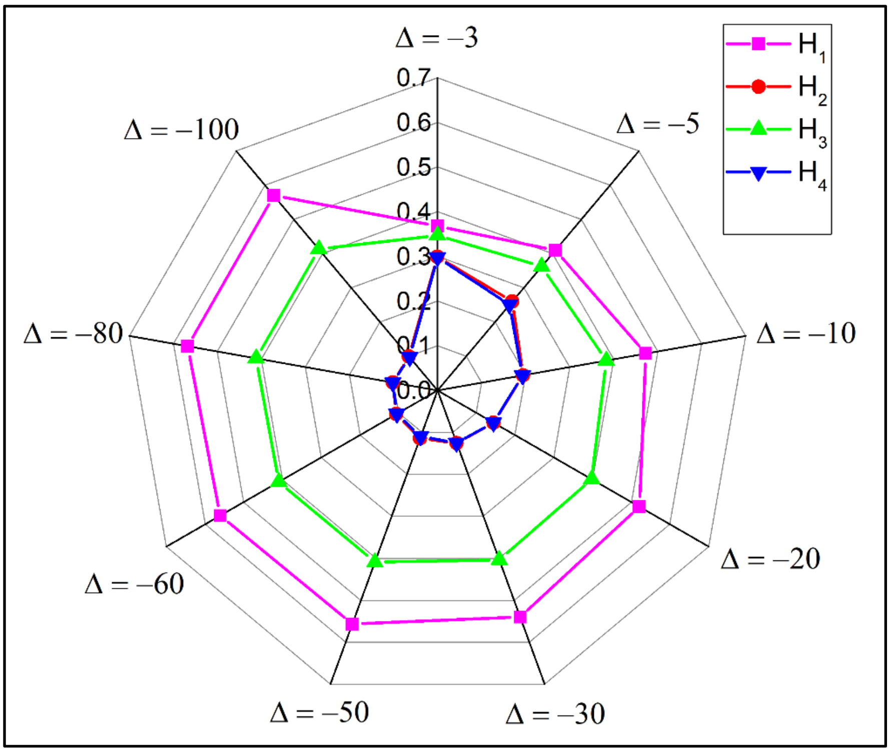

Fig. 4. Influence of Δ under the SFRSSWA operator

Similarly, in light of Table 10, it becomes evident that the score values for H_1 rise steadily from 0.3610 to 0.5690 as the values of Δ decrease, analogous to the SFRSSWA case. On the other hand, the scores of H_2 and H_4 decline from 0.3060 to 0.0980 and 0.3050 to 0.0970, respectively. Also, the score value of H_3 shows a minor increase from 0.3430 to 0.4120 as the inputs of Δ decrease. The ranking outcomes remain consistent across all values of Δ, reflecting the results of the SFRSSWG operator. The stable ranking $H_1 \succ H_3 \succ H_2 \succ H_4$ again suggests that the SFRSSWG operator is highly robust to changes in Δ certifying reliable decision support regardless of parameter tuning.

Table 10
Sensitivity of Δ under the SFRSSWG operator

Δ	**Score values**				**Ranking**
	$Sco(H_1)$	$Sco(H_2)$	$Sco(H_3)$	$Sco(H_4)$	
−3	0.3610	0.3060	0.3430	0.3050	$H_1 \succ H_3 \succ H_2 \succ H_4$
−5	0.4030	0.2660	0.3590	0.2640	$H_1 \succ H_3 \succ H_2 \succ H_4$
−10	0.4670	0.2030	0.3080	0.2010	$H_1 \succ H_3 \succ H_2 \succ H_4$
−20	0.5170	0.1520	0.3960	0.150	$H_1 \succ H_3 \succ H_2 \succ H_4$
−30	0.5380	0.1310	0.4030	0.130	$H_1 \succ H_3 \succ H_2 \succ H_4$
−50	0.5550	0.1130	0.4080	0.1120	$H_1 \succ H_3 \succ H_2 \succ H_4$
−60	0.560	0.1090	0.4090	0.1080	$H_1 \succ H_3 \succ H_2 \succ H_4$
−80	0.5650	0.1030	0.4110	0.1020	$H_1 \succ H_3 \succ H_2 \succ H_4$
−100	0.5690	0.0980	0.4120	0.0970	$H_1 \succ H_3 \succ H_2 \succ H_4$

The effect of Δ upon the ranking behavior under SFRSSWG is demonstrated in Figure 5.

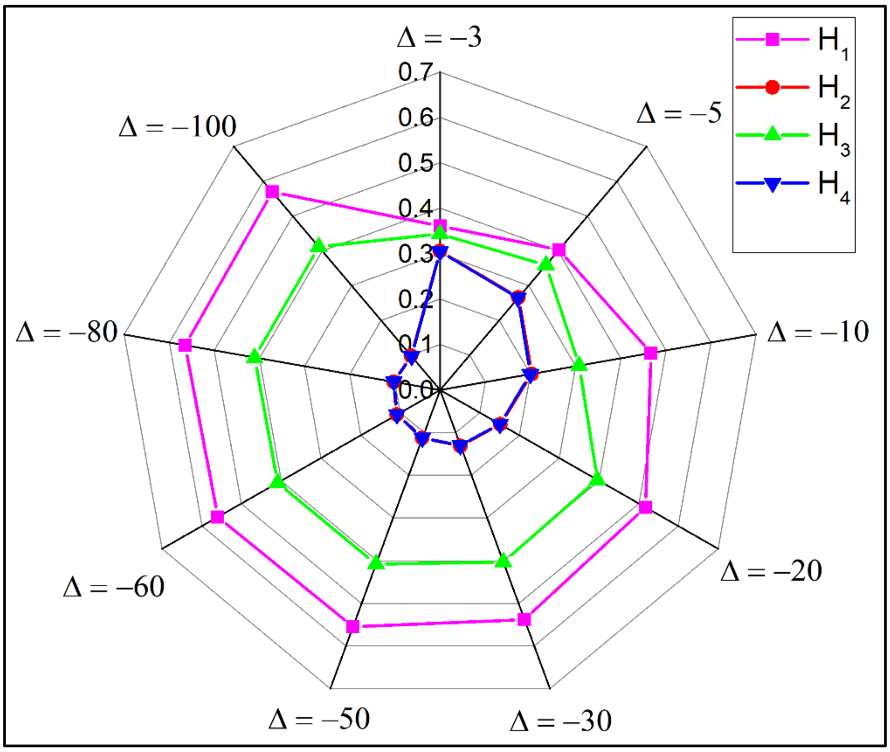

Fig. 5. Influence of Δ under the SFRSSWG operator

6. Comparative Analysis and Discussion

In this section, we present a comprehensive comparative analysis of our proposed method against several established studies, aiming to underscore the approach's efficacy and validity. The results obtained are compared with several existing methods. In [37], the authors developed a MAGDM scheme based on intuitionistic fuzzy rough SS weighted averaging (IFRSSWA) and intuitionistic fuzzy rough SS weighted geometric (IFRSSWG) operators. Sarfraz [38] articulated the spherical fuzzy SS Maclaurin symmetric mean (SFSSMSM) and the spherical fuzzy SS weighted Maclaurin symmetric mean (SFSSWMSM) operators with application in artificial intelligence. Hussain *et al.* [39] presented the Pythagorean fuzzy rough SS weighted average (PyFRSSWA) and Pythagorean fuzzy rough SS weighted geometric (PyFRSSWG) operators.

To address the aforementioned MAGDM problem, we proceed by employing the predefined weight vector $(0.21, 0.29, 0.23, 0.27)^T$ and setting the SS parameter $\Delta = -3$. The resultant score values and corresponding ranking orders attained through the proposed approach, along with those derived from existing benchmark methods, are systematically outlined in Table 11. Based on the comparative data presented in Table 11, the following key conclusions can be drawn:

i. The ranking outcomes obtained through the SFSSWMSM, PyFRSSWA, PyFRSSWG, and IFRSSWA operators are entirely aligned with those generated by the proposed SFRSSWA and SFRSSWG methods. This consistency underscores the validity and reliability of the proposed operators within the MAGDM framework.
ii. The results derived from the SFSSMSM operator exhibit a minor deviation from the ranking outcomes produced by the proposed operators. Although the overall ranking order differs slightly, specifically due to the interchange of positions between the

alternative factors H_2 and H_4, the top-ranked alternative factor remains consistent across all methods, highlighting the robustness of the best factor selection.

iii. A notable observation arises in the case of the IFRSSWG operator, where the score values of alternative factors H_2 and H_4 are equal. This reveals a potential ranking tie, which may reflect a limitation in the discrimination capability of this operator under the given circumstances.

Table 11
Comparison of different approaches with the proposed SFRSSWA and SFRSSWG operators

Operator	**Score values**				**Ranking**
	$Sco(H_1)$	$Sco(H_2)$	$Sco(H_3)$	$Sco(H_4)$	
SFRSSWA	**0.3680**	**0.2990**	**0.3470**	**0.2980**	$H_1 \succ H_3 \succ H_2 \succ H_4$
SFRSSWG	**0.3610**	**0.3060**	**0.3430**	**0.3050**	$H_1 \succ H_3 \succ H_2 \succ H_4$
SFSSWMSM [38]	**0.5340**	**0.3650**	**0.4639**	**0.3640**	$H_1 \succ H_3 \succ H_2 \succ H_3$
SFSSMSM [38]	**0.5370**	**0.3460**	**0.4630**	**0.3520**	$H_1 \succ H_3 \succ H_4 \succ H_2$
PyFRSSWA [39]	**0.3770**	**0.2674**	**0.3650**	**0.2671**	$H_1 \succ H_3 \succ H_2 \succ H_4$
PyFRSSWG [39]	**0.3870**	**0.2666**	**0.3630**	**0.2532**	$H_1 \succ H_3 \succ H_2 \succ H_4$
IFRSSWA [37]	**0.4750**	**0.3451**	**0.4723**	**0.3445**	$H_1 \succ H_3 \succ H_2 \succ H_4$
IFRSSWG [37]	**0.4890**	**0.3444**	**0.4710**	**0.3444**	$H_1 \succ H_3 \succ H_2 \approx H_4$

7. Conclusions

As the inspiration of the rural region and a main contributor to the general monetary well-being, crops are important to the stability and prosperity. Effective rules, strategic investments in crop manufacturing, and sustainable agricultural practices are crucial components that can improve economic growth, increase prosperity, and improve food security.

To determine the most suitable crop under vital standards or attributes, we investigated sophisticated approaches to the MAGDM problem in this take a look at. In this article, utilizing the idea of PAOs, SS TN, and SS TN as foundations, we proposed new AOs in the context of SFRS. These AOs consist of the SFRSSWG and SFRSSWA operators. We checked out the essential features and results of the developed operators, showcasing their adaptability and practicality.

We gave a numerical example of selecting a satisfactory crop to boost farmers' income using demonstrating the realistic application of the MAGDM approach. Additionally, the sensitivity of the proposed MAGDM approach and the associated AOs are examined concerning varying inputs of the SS parameters Δ, highlighting the robustness of the ranking results. The intrinsic flexibility of the SS TN and TCN allows the established operators to exhibit a high degree of adaptability, thereby permitting decision-makers to tailor parameter settings according to the specific demands of their DM dilemmas. Moreover, the efficacy and dependability of the evolved methodologies were shown via a comparative analysis with existing methodologies, demonstrating the superiority of our strategies.

Nevertheless, it is acknowledged that the counseling approaches might not always be enough while coping with uncertain and ambiguous statistics offered in several elements. To address those obstacles, the next studies will increase these methods to embody additional fuzzy frameworks, including complicated spherical sets, t-spherical fuzzy hypersoft set theory, and PFSs. Using those modern-day techniques, we hope to deal with real-world problems in numerous domains, which include game principles, artificial intelligence, scientific prognosis, green dealer choice, and more.

Appendix-1: Proof of Theorem 1

We demonstrate Theorem 1 in the following way by using mathematical induction.

Step 1: For $\kappa = 2$, we have:

$$\boldsymbol{SFRSSWA}(\mathrm{R}_1, \mathrm{R}_2) = \left(\begin{array}{c}\left(\sqrt{1-\left(\sum_{\sigma=1}^{2}\alpha_\sigma\left(1-\underbrace{\psi_\sigma^{\iota 2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \left(\sum_{\sigma=1}^{2}\alpha_\sigma\left(\underbrace{\lambda_\sigma^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{2}\alpha_\sigma\left(\underbrace{\delta_\sigma^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\right), \\ \left(\sqrt{1-\left(\sum_{\sigma=1}^{2}\alpha_\sigma\left(1-\overbrace{\psi_\sigma^{\mu 2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \left(\sum_{\sigma=1}^{2}\alpha_\sigma\left(\overbrace{\lambda_\sigma^{\mu}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{2}\alpha_\sigma\left(\overbrace{\delta_\sigma^{\mu}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\right)\end{array}\right),$$

which is SFRV. Thus, Eq. (22) is valid for $\kappa = 2$.

Step 2: Let Eq. (22) be true for $\kappa = \varphi$. Then:

$$\boldsymbol{SFRSSWA}(\mathrm{R}_1, \mathrm{R}_2, \cdots, \mathrm{R}_\varphi) = \left(\begin{array}{c}\left(\sqrt{1-\left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(1-\underbrace{\psi_\sigma^{\iota 2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(\underbrace{\lambda_\sigma^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(\underbrace{\delta_\sigma^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\right), \\ \left(\sqrt{1-\left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(1-\overbrace{\psi_\sigma^{\mu 2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(\overbrace{\lambda_\sigma^{\mu}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(\overbrace{\delta_\sigma^{\mu}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\right)\end{array}\right).$$

Step 3: We show that Eq. (22) is true for $\kappa = \varphi + 1$, then:

$$\boldsymbol{SFRSSWA}(\mathrm{R}_1, \mathrm{R}_2, \cdots, \mathrm{R}_\varphi, \mathrm{R}_{\varphi+1}) = \left(\begin{array}{c}\left(\begin{array}{c}\sqrt{1-\left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(1-\underbrace{\Lambda_\sigma^{\iota 2}}\right)^{\Delta}+\alpha_{\sigma+1}\left(1-\underbrace{\psi_{\sigma+1}^{\iota 2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ \left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(\underbrace{\lambda_\sigma^{\iota}}\right)^{\Delta}+\alpha_{\sigma+1}\left(\underbrace{\lambda_{\sigma+1}^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(\underbrace{\delta_\sigma^{\iota}}\right)^{\Delta}+\alpha_{\sigma+1}\left(\underbrace{\delta_{\sigma+1}^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\end{array}\right), \\ \left(\begin{array}{c}\sqrt{1-\left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(1-\overbrace{\psi_\sigma^{\mu 2}}\right)^{\Delta}+\alpha_{\sigma+1}\left(1-\overbrace{\psi_{\sigma+1}^{\iota 2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ \left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(\overbrace{\beta_\sigma^{\mu}}\right)^{\Delta}+\alpha_{\sigma+1}\left(\overbrace{\lambda_{\sigma+1}^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\varphi}\alpha_\sigma\left(\overbrace{\delta_\sigma^{\mu}}\right)^{\Delta}+\alpha_{\sigma+1}\left(\overbrace{\delta_{\sigma+1}^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\end{array}\right)\end{array}\right).$$

Then:

$$\boldsymbol{SFRSSWA}(\mathrm{R}_1, \mathrm{R}_2, \cdots, \mathrm{R}_{\varphi+1}) = \left(\begin{array}{c}\left(\sqrt{1-\left(\sum_{\sigma=1}^{\varphi+1}\alpha_\sigma\left(1-\underbrace{\psi_\sigma^{\iota 2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \left(\sum_{\sigma=1}^{\varphi+1}\alpha_\sigma\left(\underbrace{\lambda_\sigma^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\varphi+1}\alpha_\sigma\left(\underbrace{\delta_\sigma^{\iota}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\right), \\ \left(\sqrt{1-\left(\sum_{\sigma=1}^{\varphi+1}\alpha_\sigma\left(1-\overbrace{\psi_\sigma^{\mu 2}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \left(\sum_{\sigma=1}^{\varphi+1}\alpha_\sigma\left(\overbrace{\lambda_\sigma^{\mu}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\varphi+1}\alpha_\sigma\left(\overbrace{\delta_\sigma^{\mu}}\right)^{\Delta}\right)^{\frac{1}{\Delta}}\right)\end{array}\right).$$

Hence, based on the results established in *Step 1*, *Step 2*, and *Step 3*, it is apparent that Eq. (22) holds for all positive integers κ.

Theorem 1 reveals that the proposed SFRSSWA operator owns the subsequent properties.

Appendix-2: Proof of Theorem 2

Since $\mathrm{R}_\sigma = \left(\left(\underbrace{\psi_\sigma^{\iota}}, \underbrace{\lambda_\sigma^{\iota}}, \underbrace{\delta_\sigma^{\iota}}\right), \left(\overbrace{\psi_\sigma^{\mu}}, \overbrace{\lambda_\sigma^{\mu}}, \overbrace{\delta_\sigma^{\mu}}\right)\right) = \left(\left(\underbrace{\psi^{\iota 2}}, \underbrace{\lambda^{\iota 2}}, \underbrace{\delta^{\iota 2}}\right), \left(\overbrace{\psi^{\iota 2}}, \overbrace{\lambda^{\mu 2}}, \overbrace{\delta^{\mu 2}}\right)\right)$, then:

$\boldsymbol{SFRSSWA}(\mathrm{R}_1, \mathrm{R}_2, \cdots, \mathrm{R}_\kappa) = (\mathrm{R}, \mathrm{R}, \cdots, \mathrm{R})$

$$
= \left(\begin{array}{c} \left(\sqrt{1 - \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(1 - \underbrace{\psi_\sigma^{\iota 2}} \right)^{\Delta} \right)}^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(\underbrace{\lambda^{\iota}} \right)^{\Delta} \right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(\underbrace{\delta^{\iota}} \right)^{\Delta} \right)^{\frac{1}{\Delta}} \right) \\ \left(\sqrt{1 - \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(1 - \overbrace{\psi_\sigma^{\mu 2}} \right)^{\Delta} \right)}^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(\overbrace{\lambda_\sigma^{\iota}} \right)^{\Delta} \right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma \left(\overbrace{\delta_\sigma^{\iota}} \right)^{\Delta} \right)^{\frac{1}{\Delta}} \right) \end{array} \right)
$$

$$
= \left(\begin{array}{c} \left(\sqrt{1 - \left(\left(1 - \underbrace{\psi^{\iota 2}} \right)^{\Delta} \right)}^{\frac{1}{\Delta}}, \left(\left(\underbrace{\lambda^{\iota 2}} \right)^{\Delta} \right)^{\frac{1}{\Delta}}, \left(\left(\underbrace{\delta^{\iota 2}} \right)^{\Delta} \right)^{\frac{1}{\Delta}} \right) \\ \left(\sqrt{1 - \left(\left(1 - \overbrace{\psi^{\iota 2}} \right)^{\Delta} \right)}^{\frac{1}{\Delta}}, \left(\left(\overbrace{\lambda^{\mu 2}} \right)^{\Delta} \right)^{\frac{1}{\Delta}}, \left(\left(\overbrace{\delta^{\mu 2}} \right)^{\Delta} \right)^{\frac{1}{\Delta}} \right) \end{array} \right) = \left((\psi^{\iota 2}, \lambda^{\iota 2}, \delta^{\iota 2}), (\psi^{\mu 2}, \lambda^{\mu 2}, \delta^{\mu 2}) \right) = \mathrm{R}.
$$

Conflicts of Interest
The author declares no conflicts of interest.

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